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The saturation of household spending diversity and emergent properties of representative households

Andreas Chai[*], Christian Kiedaisch[†], Nicholas Rohde[‡]

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Abstract

This paper studies the diversity of household consumption spending, i.e. how widely households distribute their spending across different types of goods. Using detailed expenditure data from the UK (1990 - 2015), we show that the diversity of household spending rises in income up to a certain level and then starts to decline as richer households increasingly concentrate their spending on specific expenditure categories. As these specific expenditure categories differ across households, spending diversity on the aggregate level can keep rising in income while spending diversity on the household level falls. This implies that the process of aggregation, which is often used to infer the behavior of representative (average) households, generates emergent properties, i.e. trends that only exist on the aggregate level but not on the level of individual households. We build a model with heterogeneous non-homothetic preferences that can explain these observed patterns. Our model shows that ignoring preference heterogeneity and assuming representative households leads to a (potentially very large) underestimation of the value of product variety.

JEL classification: D12, D11, C14, O33.

Keywords: Spending Diversity, Engel's Law, Emergent Properties, Demand for Variety, Aggregation.

[*]Griffith Business School, Gold Coast Campus Griffith University, Australia QLD 4222. a.chai@griffith.edu.au.

[†]Corresponding author: CERPE-DeFiPP, University of Namur, Belgium, christian.kiedaisch@unamur.be

[‡]Griffith Business School, Gold Coast Campus Griffith University, Australia QLD 4222. n.rohde@griffith.edu.au

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'Important dimensions of heterogeneity and diversity that are masked in macro data were uncovered... Inspection of cross section data reveals that otherwise observationally identical people make different choices... Inspection of these data reveals the inadequacy of the traditional representative agent paradigm...' *James Heckman, Bank of Sweden Nobel Memorial Lecture in Economic Sciences, December 8, 2000*

1 Introduction

One of the most salient features of developed economies is the wide variety of goods and services that their inhabitants consume. The demand for these goods depends on income but can also be very heterogeneous across consumers with the same income levels, as indicated in the above quote (see also Blundell and Stoker (2005)). This paper considers how the tendency for consumers to diversify their spending across different goods changes with income (Theil and Finke, 1983; Clements et al., 2006; Clements and Gao, 2012). We show that the relationship between spending diversity and income is conditional on the level of aggregation, in that certain patterns are only visible on an aggregate level. Taking these observations into account, we analyze how using representative agent models can generate biases in welfare estimates.

Ever since the discovery of Engel's Law (Engel, 1857), it has been recognized that the way in which income affects the distribution of household expenditures across goods has broad economic consequences. Changes in the composition of household spending induced by income changes can stimulate structural change (Pasinetti, 1981; Saviotti, 2001; Metcalfe et al., 2006; Herrendorf et al., 2014), affect innovation (Föllmi and Zweimuller, 2006; Jaravel, 2018) and influence international trade flows (Markusen, 2013; Fajgelbaum et al., 2011; Föllmi et al., 2018). A number of researchers have begun considering how behavioural heterogeneity can be modelled (Calvet and Common, 2003; Beckert and Blundell, 2008) and many recognize that it is crucial to study the precise relationship between aggregate and individuals behavior (Grandmont, 1987, 1992; Hildenbrand, 1994; Quah, 1997; Blundell and Stoker, 2005). Our paper represents a contribution to this literature.

Using UK household level spending data (1990 to 2015), we analyze the relationship between spending diversity (measured by the entropy of consumption shares (Theil, 1967)) and income, which we dub the "Engel curve for spending diversity". We find the following Stylized Fact: there is an inverse-U relation between spending diversity and income at the level of individual households. This means that a rise in income induces poor households to allocate their spending more evenly across different goods, while it prompts relatively rich households to re-concentrate their spending on particular goods, so that there is a decline of spending diversity in the latter case. We find this result to be robust across a number of years. It represents an empirical regularity that, similar to Engel's Law, describes how the composition of household expenditure evolves with income.

We then examine the extent to which a similar pattern of spending diversity is evident at more aggregated levels of household spending. We do so by aggregating spending data for groups of households with similar income levels. We then estimate the diversity of spending for these groups of households. We also derive the Engel curve for spending diversity at this aggregate level. We observe the following Stylized Fact: spending diversity at the aggregate level either always rises in income, or it only starts to fall in income at larger income levels than individual spending diversities do. Therefore, there is an income range for which individual household spending diversity falls while spending diversity at the aggregate level paradoxically rises. The pattern at the aggregate level is therefore 'emergent' in the sense that it does not reflect the pattern found at the individual household level and seems to result from the process of aggregating spending patterns across households (see also Hildenbrand (1994); Blundell and Stoker (2005)).

As spending diversity at the individual household and at the aggregate level can only differ when there is heterogeneity in spending patterns across households, we explain our results by the fact that individual households concentrate their spending on different goods when their income rises. This implies that aggregation across households masks the decline of spending diversity observed at the individual household level. This explanation is in line with the empirical regularity that Engel curves are highly heteroscedastic, i.e. that the quantities of a particular good that are consumed by different households become more distinct when household income grows (Blundell and Stoker, 2005; Lewbel, 2008).

The finding that the relationship between spending diversity and income observed at the aggregate level does not mirror that observed at the individual household level sheds new light on studies by Theil and Finke (1983), Clements and Chen (1996), and Clements et al. (2006). Using aggregate cross-country data, these papers found that the spending diversity observed at this level of aggregation always rises in income, and have interpreted this as evidence that individual households seek to allocate their expenditures more evenly across different goods and services when their income grows. Using more disaggregated data from multiple waves of the UK household expenditure survey, our analysis shows that this need not be the case as individual household spending diversity can fall in income even though spending diversity on the aggregate level rises.[1]

In order to explain the empirical patterns observed at both the household and aggregate level, we build a model in which households have non-homothetic, heterogeneous preferences. Non-homothetic preferences are needed in order generate income dependent consumption shares that result in income dependent spending diversity. This is consistent with the empirical observations that the budget share for food falls in income ("Engel's Law", Engel (1857); Lewbel (2008); Chai and Moneta (2010)) and that the budget shares of other items belonging into the broader categories of goods and services tend to rise in income. Preference heterogeneity is needed in order to obtain heteroscedastic Engel curves (as observed in the data) and in order to generate differences between spending diversity at the household and aggregate level.

We show that our model can explain all the Stylized Facts that we observe in the data. We then use the model to assess the pitfalls of making the simplifying assumption that every household has the same (average) preferences, i.e. that there exists a representative household.[2] We consider the stylized case in which - keeping the size of the population and the distribution of income fixed - a model with representative households yields the same aggregated (market) demand for each good as a more general model, in which households have heterogeneous preferences. We then analyze how much an increase in product variety is valued by individual households with heterogeneous preferences and by representative households with average preferences. We find that individual households with heterogeneous preferences (all) value an increase in product variety more than representative households do and that these welfare differences can become arbitrarily large for certain parameter values.[3] Our results therefore show that standard representative household models can substantially underestimate the welfare gains of policies that increase the variety of available goods (like innova-

[1]In a preliminary study, Chai et al. (2015) examined a specific year (2001) and aggregation level (12 categories) and also found that the relation between individual household spending diversity and income can be inverse U-shaped in the UK household expenditure data.

[2]A critical discussion of the representative household assumption is given by Kirman (1992).

[3]When preferences are instead homothetic, we find that preference heterogeneity does not affect the value of product variety. While the simplifying assumption of homothetic preferences is often made in product-variety models (based on Dixit and Stiglitz (1977)), it is fairly unrealistic as it implies that the income shares devoted to different goods are independent of household income.

tion, antitrust, e-commerce or trade policies).[4] As GDP growth is in part driven by the introduction of new goods, our results therefore imply that ignoring preference heterogeneity leads to an underestimation of the growth rate of real GDP (due to an overestimation of the rise of the price index).

Jerison (1984) and Dow and Werlang (1988) have already shown that "positive" representative households the preferences of which allow to derive the aggregated (market) demand for each good can be Pareto inconsistent, and can prefer a situation B to A even though all the households in the represented economy prefer A to B. Jerison (2016) studies this question in more detail and analyzes how large the Pareto-inefficiencies can become. In various examples that for example study the welfare effects of the doubling of a certain price, he finds that the Pareto-inefficiencies tend to be small (unless Giffen goods - which are absent in our model - are involved). Unlike the present paper, he, however, does not analyze the effects that consumer heterogeneity might have on the value of product variety.

A closely related paper is Neiman and Vavra (2021) who use data on US non-service retail spending from 2004 - 2016 and also study differences between individual and aggregate level spending patterns. They find that individual spending diversity fell over time while spending diversity on the aggregate level rose, and that these movements can be explained by the facts that households increasingly concentrated their spending on fewer goods and increasingly purchased different products from each other.[5]

Neiman and Vavra (2021) develop a model in which households have CES preferences and face fixed costs per variety consumed due to which they only consume positive quantities of a subset of the available goods.[6] As households are assumed to have heterogeneous preferences with respect to specific goods, these subsets differ across households. The authors show that within their model an increase in product variety mainly increases welfare due to the fact that it permits households to consume subsets

[4]The gains from an increase in product variety resulting from trade are for example studied by Broda and Weinstein (2006) and Arkolakis et al. (2012), who both use representative agent models with CES preferences. Various papers (like Hausman (1997) and Brynjolfsson and Smith (2003)) estimate the value of increased product variety by analyzing the compensating variation associated with the drop of a product's price from its "virtual price", at which there is no demand for the product, to its current price. These estimates are based on parameters derived from market- and not individual demand curves. As our analysis shows that preference heterogeneity can strongly affects the value of product variety even when market demand curves are unchanged, the estimates from the above-mentioned papers therefore do not appropriately reflect the welfare of individual households when preferences are heterogeneous as modeled in this paper.

[5]In this paper, we refrain from studying household spending patterns over time as it is difficult to control for exogenous changes in variety demand and for changes in the number of available goods over time.

[6]In our setting, consumption zeros can occur even without such fixed costs as we allow for more general non-homothetic preferences.

of products that are better suited to their particular tastes. In our welfare analysis, we instead focus on the case in which households always consume positive quantities of all existing goods, and in which preference heterogeneity affects the intensive, but not the extensive consumption margins. In our setting, preference heterogeneity only affects the value of variety when preferences are (as implied by the data) non-homothetic, but not when they are of the CES type, as assumed in Neiman and Vavra (2021).

The paper is structured as follows: Section 2 presents Stylized Facts about household spending diversity that we observe in the data. Section 3 presents our model, which is then used for the welfare analysis in Section 4. Section 5 concludes. Tables, figures, and proofs are presented in the Appendix.

2 Stylized Facts about spending diversity

We begin by highlighting some Stylized Facts about how households diversify their spending across different goods, and about how this diversity changes with income. We do this by estimating the Engel curve for spending diversity. We estimate these Engel curves both at the household level and at a more aggregated level for groups of households that possess similar incomes. The latter is done in order to characterize the average behavior of "representative households". This allows to investigate how the aggregation process can generate emergent properties in spending patterns, i.e. properties that do not exist at the levels of actual households, but only emerge at the aggregate level.

We use the following notation in our analysis: There are n households indexed by i and k expenditure categories (or goods) indexed by j. Total expenditures on all k categories by household i are denoted by x_i (and also referred to as income). The expenditure share of household i on good j is denoted by s_{ij}, so that $s_i = (s_{i1}, s_{i2}..., s_{ik})$ denotes the vector of expenditure shares for household i. The overall expenditures of household i on a good j are consequently given by $x_i \times s_{ij}$.

We measure the diversity of household spending across expenditure categories by the entropy of the shares. While there exist a number of different diversity measures that can be used for this purpose,[7] we follow Theil (1967), Theil and Finke (1983), and

[7]There are several other measures of spending diversity, like the Gini-Simpson and the Hirschmann-Herfindahl index. In Figures 19 - 23 in the appendix it is shown that our empirical results are robust if these other measures are used (see also Chai et al. (2015)).

Clements et al. (2006) and use the following:

$$E_i = -\sum_{j=1}^{k} \phi(s_{ij}) \qquad \begin{cases} \phi(s_{ij}) = s_{ij} \ln s_{ij} & s_{ij} > 0 \\ \phi(s_{ij}) = 0 & s_{ij} = 0 \end{cases} \tag{1}$$

The entropy E_i is a number that measures the extent to which spending of household i is dispersed across expenditure categories. It takes on a value of zero when all the expenditure is concentrated on a single item, and is equal to $\ln(k)$ (> 0) when the shares on all items are equal. We use this measure to estimate the cross-sectional household level Engel curve for spending diversity, i.e. the relationship between E_i and x_i.

In order to study the behavior of representative consumers, we sort our sample of households according to their expenditure levels ($x_1 < x_2 < ...x_n$) and partition them into 50 income groups.[8] The expenditure shares are then averaged within these groups in order to derive a measure of spending diversity at the aggregate (average) level. To do so, the average shares at the group level are denoted $\hat{s}_{jd} = [50/n] \sum_{i \in d} s_{ij}$, where d is the group under consideration.[9] The entropy $\hat{E}$ of these shares $\hat{E}_d(\hat{s}_{jd})$ is then calculated as a function of the average income level of households within a group and denoted as the spending diversity at the (aggregate) group level. This allows the Engel curve for spending diversity on the aggregate level, i.e. the relationship between $\hat{E}$ and x, to be derived. As the distributions of aggregate expenditures within the richer (poorer) groups are likely to be similar to those in richer (poorer) countries, we can compare our results to those derived in the cross country studies discusses above (Theil and Finke, 1983; Clements and Chen, 1996; Clements et al., 2006).

To depict the Engel curves for spending diversity on the individual and aggregate level, we use kernel regressions based on Nadarya (1964) and Watson (1964). These are non-parametric regressions for which it is not necessary to assume a specific functional form for the relationship between E and x. We use second order polynomial terms and choose the bandwidth that minimizes the mean integrated squared error.

In terms of data, we use annual household data sourced from the UK Family Expendi-

[8] Figure 13 shows that the results do not change much when households are instead partitioned into 20 or 10 income groups.

[9] As we consider households' equivalence-scale-adjusted expenditures x_i and not their actual expenditures, we base our analysis on the average of the budget shares of all households within a group (i.e. on $\hat{s}_{jd} = [50/n] \sum_{i \in d} s_{ij}$), instead of basing it on the share of the total (non-equivalence-scale-adjusted) expenditures on good j by households falling into group d.

ture Survey (FES) from 1990 to 2015. Over this time period, the classification method for expenditure categories has been subject to some change. To ensure consistency across sample periods, the classification method specified by the Office of National Statistics in 2000 featuring $k = 12$ categories (see Table 1) was selected and retrospectively applied to the data. In addition, we also study the case of three categories in which the 12 categories are aggregated into 'Food', 'Goods' and 'Services', and the case of 200+ aggregation categories in which no aggregation procedure is used.

We exclude certain housing expenditures because of well-known problems with this data (Tanner, 1999; Blow et al., 2004). Savings are also excluded as we focus on consumption expenditures. We censor the data by removing Northern Ireland and households with more than two adults, but keep all households with two adults and any number of children.[10] In order to control for differences in the size of households, OECD equivalence scales are used.

Household spending on major durable spending items (e.g. automobile purchases) is converted into weekly expenditure equivalents as provided by the UK Office for statistics. Household income x_i is calculated as the sum of the expenditures (and expenditure equivalents) on all expenditure categories that are included in the analysis (and consequently excludes savings and certain housing expenditures). Inflation is accounted for by using the Retail Price Index (RPI) percentage change over 12 months.[11] In terms of the growth rate of total expenditure, our data is broadly consistent with other data sets devised by Blow et al. (2004) and the UK National Accounts.[12] Across the observed period, the average annual sample size is about 6000 observations but drops to 5000 between 1998 to 2000. The years 1990, 1995, 2000, 2005, 2010 and 2015 were selected in order to study spending patterns across a significant time span (25 years).

As most household expenditure surveys have less observations at high levels of household income, a common problem is sample selection bias. However, Tanner (1999) finds that the ratio of non-housing total expenditure in the FES to non-housing total expenditure in the National Accounts was around 90 per cent between 1974 and 1992. More recently, Blundell and Etheridge (2010) noted that the coverage of the FES

[10] This reduces the number of share houses and households co-inhabited by extended family in the sample.

[11] This is calculated using data from the UK Office of National statistics on all consumption items except for housing and mortgage payments (CDKG).

[12] Some differences are likely due to the fact that we have dropped households with more than two adults and excluded recall categories from 1986.

has been consistent, rarely dropping below 90 per cent of the national accounts level. This instills us with some confidence that the magnitude of the sample selection bias is not too large as the FES expenditure match the National accounts relatively well. We moreover remove all households with incomes more than three standard deviations above the average household income.

Table 2 provides an overview of how the average subgroup budget shares $\hat{s}_{jd}$ for the three broad categories of Food, Goods, and Services evolve across both income levels and time (for simplicity, the case of 10 subgroups is considered). Income x is measured by the group average of the real weekly total expenditure. This table reveals a relatively stable pattern that is consistent with Engel's Law: as household income rises, the average budget share dedicated to food declines. Also consistent with other studies is the fact that poor households on average spend a considerable fraction of their budget on food (Banerjee and Duflo, 2007; Clements et al., 2006), while spending on average tends to become more widely dispersed across different expenditure categories when income rises.

Table 3 reports that there is a considerable degree of heteroscedasticity in the relation between the expenditures of a household on a particular consumption item and household income, even if we control for a range of demographic variables. The heterogeneity in preferences and demand, defined as the variation in household spending that cannot be explained by income or other household characteristics, is therefore significantly and positively correlated with household income. This result is in line with several previous studies (Banks et al., 1997; Blundell and Stoker, 2005; Calvet and Common, 2003).

Figures 1 and 2 show the relation between the total variety of goods consumed by a household (i.e. the number of items of which a household consumes positive quantities) and household income for the case of 12 and 200+ consumption categories. The Figures show that the variety of goods consumed increases strongly in income for low income levels but then stabilizes above a certain income level. A positive relation between income and consumption variety has also been observed in previous studies (Jackson, 1984; Falkinger and Zweimüller, 1996).

In the following, we present the main empirical results of our analysis.

FIGURE 3 ABOUT HERE

Figure 3 depicts the estimated Engel curves for spending diversity observed on the individual (household) level (left hand side), as well as on the group level (right hand side). Note that income is measured by real weekly total household expenditure. The first row depicts the case where consumption items are aggregated into three broad categories (food, goods and services), the middle row depicts the 12 good aggregation (see Table 1), and the last row the case where goods are highly disaggregated (200+ expenditure categories). Each figure contains curves for six years from 1990 to 2015.

From these results, a number of Stylized Facts can be observed:

- **Stylized Fact 1**: There exists an inverse-U relationship between spending diversity observed on the household level, E_i, and household income x.

At low income levels, the Engel curve for spending diversity $E_i(x_i)$ has a positive slope, implying that households allocate their spending more evenly across goods when their income rises. At high income levels, E_i, however, tends to fall in income as the opposite is the case: households tend to concentrate their spending more on particular consumption categories when their income grows.

- **Stylized Fact 2**: On the more aggregated group level, the Engel curve for spending diversity is either upward sloping or has an inverse-U shape. There is therefore either a positive relation or an inverse-U relation between the diversity (entropy) $\hat{E}$ of (aggregated) group spending and average group income x.

Interestingly, relative to spending diversity on the aggregate level, spending diversity on the individual household level falls more rapidly in income at high income levels in the case of 200+ expenditure categories.

FIGURES 4, 7, and 10 ABOUT HERE

The choice of the aggregation level (household versus group level) thus seems to impact the levels of spending diversity and the shapes of the Engel curves for spending diversity. This can be seen more clearly in Figure 4 where E_i and $\hat{E}$ are depicted together for the different years in the case of three expenditure categories. From this figure, as well as from Figures 7 and 10 that consider the cases of 12 and 200+ expenditure categories, the following Stylized Fact emerges:

- **Stylized Fact 3**: The Engel curve for spending diversity on the aggregate (group) level is always situated above the Engel curve for spending diversity on the individual (household) level. In other words, $\hat{E}$ exceeds E_i for each level of household income x.

Spending diversity on the group level is therefore greater than spending diversity observed on the individual level across all income levels. This suggests that the process of aggregating household expenditure leads to an increase in observed spending diversity. If each household with a certain income x would spend its income in exactly the same fashion, then $\hat{E} = E_i$ would hold. The observed pattern must therefore stem from the fact that different households belonging to the same income groups allocate their spending differently across different goods. As such, these observed differences between $\hat{E}$ and E_i represent a measure of differences in household spending patterns.

An interesting pattern of the data is that the entropy $\hat{E}$ of household spending on the group level appears to keep rising in income at income levels at which individual household entropy E_i already falls in income. $\hat{E}$ consequently reaches its maximum (in case there is one) at higher levels of income than E_i does.

FIGURES 5, 8, and 11 ABOUT HERE

To investigate this pattern further, Figure 5 shows the difference between aggregate and household level spending diversity ($\hat{E} - E_i$) for the case of three consumption categories in each year. Figures 8 and 11 show the same for the cases of 12 and 200+ categories. These Figures confirm that these entropy differences tend to increase in income when income is large and also show that they tend to fall in income for low income levels.[13] But are these pattern statistically significant? Figures 6, 9, and 12 report the estimated derivative $\frac{\partial(\hat{E}-E_i)}{\partial x}$ with 95% confidence intervals for the cases of 3, 12 and 200+ expenditure categories across all years. These graphs show that the derivative $\frac{\partial(\hat{E}-E_i)}{\partial x}$ is in the majority of the cases significantly positive at high income levels. In contrast, at low income levels, the derivative tends to be significantly negative. This implies that the entropy difference $\hat{E} - E_i$ tends to significantly rise in income at high income levels and that it tends to significantly fall in income at low income levels. From the Figures mentioned above, we obtain our last Stylized Fact:

- **Stylized Fact 4**: The difference $\hat{E} - E_i$ between the spending diversity on the (aggregate) group and the household level is U-shaped in the income dimension or (exceptionally) always rises in income.

[13] With the exception that entropy differences always rise in income in 1990 in the case of 200+ categories

This suggests that the heterogeneity in variety demand across households belonging to the same income group depends on the level of income and that it rises in income when income is sufficiently large. As can be inferred from Figures 5, 8, and 11, this Stylized Fact results from the following shapes of the entropy curves: at low income levels, both $\hat{E}$ and E_i rise and $\hat{E} - E_i$ tends to fall. At high levels of income, we observe the reverse: household spending diversity E_i falls, while $\hat{E}$ either rises or falls less strongly, implying that $\hat{E} - E_i$ increases.

It should be noted that, unlike in Figure 3, the E_i curves are shortened to the length of the $\hat{E}$ curves in Figures 4 to 12. In these Figures, both curves therefore begin at the average income of the poorest of the 50 income groups and end at the average income of the richest of these groups, as those are the values for which the $\hat{E}$ curve is properly defined.[14] In Figure 13, the Engel curves for spending diversity on the aggregate level (i.e. $\hat{E}$ as a function of x) are plotted for the cases where households are grouped into 10 groups (left), 20 groups (middle), and 50 groups (right) and where averages are formed within these larger groups (the case of three consumption categories is considered). The Engel curves for spending diversity can then only be plotted for a smaller income range, but their shapes do not change much within this range.

3 Model setup

We now turn to introduce a model that can account for the Stylized Facts relating to the Engel curves for spending diversity on the individual (household) and aggregate (group) level, as well as the observed differences between these two curves. We then use the model in order to undertake a welfare analysis.

The utility of household i is given by the generalized Stone Geary form:

$$U_i = \left[\sum_{j=1}^{k} \beta_{ij}^{\frac{1}{\varepsilon}} \left(q_{ij} - \gamma_j \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \tag{2}$$

The terms $q_{ij} \geq 0$ denote the quantity of good j consumed by household i and $\gamma_j \gtreqless 0$ the "subsistence consumption" level of good j. This utility function is only defined if $q_{ij} \geq \gamma_j$ holds, i.e. if the household is rich enough to consume the subsistence level of

[14]We refrain from artificially extending this curve to lower and higher values of x in order to avoid that for the lowest and highest values of x, E_i "mechanically" falls short of $\hat{E}$ simply due to the fact that E_i rises (falls) in x when x is small (large) and that this trend is averaged out in the $\hat{E}$ curve.

all goods with $\gamma_j > 0$. The parameter $\varepsilon > 0$ determines the degree of substitutability between goods: when $\varepsilon \to 0$, goods become perfectly complementary (utility is then given by $\underset{\varepsilon \to 0}{lim} U_i = \underset{j}{min} \{\beta_{ij}(q_{ij} - \gamma_j)\}$), while they become perfectly substitutable when $\varepsilon \to \infty$. When $\varepsilon = 1$, utility is given by the standard Stone-Geary form: $U_i = \prod_{j=1}^{k} (q_{ij} - \gamma_j)^{\beta_{ij}}$. The degree of substitutability therefore increases in ε.

It is assumed that $\sum_{j=1}^{k} \beta_{ij} = 1$ holds. In order to explain the empirically observed heterogeneity of consumption patterns of households with similar incomes, some preference heterogeneity is introduced: It turns out that all patterns observed in the data can be explained by assuming that only the parameters $\beta_{ij} \geq 0$ can vary across households, while the parameters γ_j are the same for all of them. It is therefore assumed that the subsistence consumption levels γ_j are the same for all households (as they might reflect "biological" needs for food, shelter etc.), while households might differ with respect to the relative importance that they attribute to consumption exceeding these levels (that is reflected by the size of the parameters β_{ij}).

Total income (or expenditures) of household i is denoted by x_i and the price of one unit of good j by p_j. The budget constraint of household i is therefore given by:[15]

$$x_i = \sum_{j=1}^{k} p_j q_{ij} \tag{3}$$

The following analysis focuses on the case in which the income of household i lies (weakly) above the threshold income level $\underline{x}$ which is required to purchase positive quantities of all goods ($q_{ij} > 0$), i.e. in which $x_i \geq \underline{x}$ (**Condition A**) holds.[16] Setting up the Lagrangian $L_i = U_i + \lambda_i \left[x_i - \sum_{j=1}^{k} p_j q_{ij} \right]$ and deriving with respect to q_{ij} gives the first order conditions:

$$\frac{\partial L_i}{\partial q_{ij}} = U_i^{\frac{1}{\varepsilon}} \beta_{ij}^{\frac{1}{\varepsilon}} (q_{ij} - \gamma_j)^{-\frac{1}{\varepsilon}} - \lambda_i p_j = 0 \tag{4}$$

Dividing the first order conditions for goods j and $l \neq j$ by each other gives the equation:

$$\frac{q_{ij} - \gamma_j}{(q_{il} - \gamma_l)} = \frac{\beta_{ij}}{\beta_{il}} \left(\frac{p_l}{p_j} \right)^{\varepsilon} \tag{5}$$

[15] As utility is strictly increasing in q_{ij}, this budget constraint is always satisfied with equality.

[16] In the case where $\gamma_j \geq 0 \; \forall j$, Condition A is given by $x_i > \sum_{j=1}^{k} p_j \gamma_j = \underline{x}$.

Combining equations 5 and 3 then allows us to solve for the optimal quantity q_{ij}^* of good j by household i:[17]

$$q_{ij}^* = \frac{x_i - \sum_{l \neq j} \left[p_l \gamma_l - p_l \gamma_j \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} \right]}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon}} \tag{6}$$

The optimal quantity q_{ij}^* is a linear function of income x_i, implying linear Engel curves. As q_{ij} increases by $\frac{1}{p_j + \sum_{l \neq j} p_l \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon}}$ $(= \frac{\beta_{ij}}{p_j}$ if $\varepsilon = 1)$ for each unit that x_i increases, the slope of the Engel curve for good j increases in β_{ij} and decreases in p_j. Differences in the taste parameters β_{ij} across households can therefore generate the heteroscedasticity of Engel curves that is observed in the data.[18]

The Engel curve of household i for good j shifts up when γ_j increases and the size of this shift does not depend on x_i or γ_l $(l \neq j)$.[19] The income elasticity of demand for good j by individual i is given by

$$\epsilon_{jx}(i) = \frac{\partial q_{ij}^*}{\partial x_i} \frac{x_i}{q_{ij}^*} = \frac{x_i}{x_i - \sum_{l \neq j} \left[p_l \gamma_l - p_l \gamma_j \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} \right]} > 0 \tag{7}$$

and therefore decreases if γ_j increases. Goods with a high value γ_j therefore represent basic need goods on which poor households concentrate their expenditures, while goods with a lower (or even negative) value γ_j are more luxurious and are only purchased in substantive amounts by rich households. The share of the budget that household i allocates to good j is given by $s_{ij} = \frac{q_{ij}^* p_j}{x_i}$ and increases in β_{ij} as q_{ij}^* increases in β_{ij} if Condition A holds (as a strict inequality).

3.1 An example with three goods

In order to show which mechanisms can generate the four Stylized Facts observed in the data, the following simple example is considered: There are three goods consisting of one basic need good $j = 1$ for which $\gamma_1 > 0$ holds and two more luxurious goods

[17] Equation 5 can be rewritten as $q_{il} = \gamma_l + \frac{\beta_{il}}{\beta_{ij}} \left(\frac{p_j}{p_l} \right)^{\varepsilon} (q_{ij} - \gamma_j)$ and equation 3 as $q_{ij} = \frac{x_i - \sum_{l \neq j} p_l q_{il}}{p_j}$. Inserting the first into the latter then gives the result. The resulting quantities are optimal as the second order conditions are satisfied.

[18] Another way to generate such heteroscedasticity within the model setup would be to assume that different households face different prices p_{ij} for the same goods j. We, however, focus on the case of preference heterogeneity which we believe to be an important driver of this empirically observed heteroscedasticity.

[19] While the Engel curve of household i is only defined for $x_i \geq \underline{x}$ (i.e. when $q_{ij}^* \geq 0$), they all "originate" at the values $q_{ij} = \gamma_j \gtrless 0$ that are reached when $x_i = \sum_{j=1}^{k} p_j \gamma_j$ holds. See Figure 16 (right hand side) for the case of three goods.

$j = 2$ and $j = 3$, for which $\gamma_2 = \gamma_3 \gtreqless 0$ holds. While the price of Good 1 is normalized to one ($p_1 = 1$), the prices of Goods 2 and 3 are given by $p_2 = p_3 = p$. While β_{i1} (the welfare weight on Good 1) is assumed to be the same for all households and equal to the constant $\beta_{i1} = 1 - \bar{\beta}$, the degree to which household i prefers Good 2 over Good 3 is allowed to vary within the range in which the household still purchases positive quantities of all available goods and in which $\beta_{i2} + \beta_{i3} = \bar{\beta}$ holds.

From equation 6 we can infer that q_{i1} and also the sum $q_{i2} + q_{i3}$ then only depend on the aggregate welfare weight $\bar{\beta}$ for Goods 2 and 3, but not on how much Goods 2 and 3 are liked by a particular household. This allows us to study the role of individual heterogeneity in the following simple setup:

There are two households: that of Mr Brown ($i = B$) and that of Mrs Jones ($i = J$). Both households are assumed to have the same income $x_i = x$, but have opposing preferences with respect to the otherwise identical Goods 2 and 3, so that $\beta_{B2} = \beta_{J3}$ and $\beta_{B3} = \beta_{J2}$ holds in addition to $\beta_{i2}+\beta_{i3} = \bar{\beta}$ (implying that $\beta_{B2}+\beta_{J2} = \beta_{B3}+\beta_{J3} = \bar{\beta}$). The aggregated (market) demand $Q_j = q_{Bj} + q_{Jj}$ for Goods $j = 1$, $j = 2$ and $j = 3$ then only depends on x, γ_j, p and $\bar{\beta}$, but not on the individual values β_{i2} and β_{i3}, as individual preference heterogeneity washes out in the aggregate. Aggregated demand for Good j and also the elasticity of aggregated demand with respect to the relative price p is therefore the same as in the case where both households value both goods equally ($\beta_{i2} = \beta_{i3} = \frac{\bar{\beta}}{2}$), and can also be derived from the utility maximization problem of two "average" households with preference parameters $\beta_{a1} = 1 - \bar{\beta}$ and $\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$ and (per household) expenditures $x_a = x$.[20]

Using equation 6 and the parameter assumptions from above, the optimal budget shares can be derived as

$$s_{B1}(x) = s_{J1}(x) = \frac{q^*_{i1}(x)}{x} = \frac{\left(1-\bar{\beta}\right)\left(x - 2p\gamma_2\right)p^{\varepsilon} + \gamma_1\bar{\beta}p}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{8}$$

$$s_{B2}(x) = s_{J3}(x) = \frac{pq^*_{B2}(x)}{x} = \frac{p\left[\beta_{B2}\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{9}$$

$$s_{B3}(x) = s_{J2}(x) = \frac{pq^*_{B3}(x)}{x} = \frac{p\left[\left(\bar{\beta} - \beta_{B2}\right)\left(x - \gamma_1 - 2p\gamma_2\right) + \left(1-\bar{\beta}\right)\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2\right]}{x\left(p\bar{\beta} + \left(1-\bar{\beta}\right)p^{\varepsilon}\right)} \tag{10}$$

[20] The analysis would be similar in a setting with more than two households as long as there was an equal number of households of each type. As households of the same preference type would then have the same linear Engel curves (assuming that $x_i > \underline{x}$ holds), only the total income of each group would then matter for aggregated demand and not its distribution across households of the same preference type.

Without loss of generality, it is assumed that $\beta_{B2} = \beta_{J3} > \beta_{B3} = \beta_{J2}$ holds, implying that $s_{B2}(x) = s_{J3}(x) > s_{B3}(x) = s_{J2}(x)$ if $x > \underline{x}$, i.e. that household Brown prefers Good 2 over Good 3, while household Jones prefers Good 3 over Good 2. A graphical representation of the Engel curves resulting in this 3-good example is given in **Figure 16**.

The level of household spending diversity $E_i(x)$ is measured by the entropy of consumption spending of households. This is given by

$$E_B(x) = -s_{B1}\ln s_{B1} - s_{B2}\ln s_{B2} - s_{B3}\ln s_{B3} = E_J(x) = -s_{J1}\ln s_{J1} - s_{J2}\ln s_{J2} - s_{J3}\ln s_{J3} \tag{11}$$

For a given income level x, this entropy is therefore the same for both households as their consumption shares coincide for Good 1, and are simply reversed for Goods 2 and 3.

When aggregated household spending is considered, the share $\hat{s}_1(x) = s_{B1}(x) = s_{J1}(x)$ of the aggregated income (which equals $2x$) is spent on Good 1 and the shares $\hat{s}_2(x) = \hat{s}_3(x) = \frac{p\left(q^*_{B2}(x)+q^*_{J2}(x)\right)}{2x} = \frac{s_{B2}(x)+s_{J2}(x)}{2} = \frac{s_{B2}(x)+s_{B3}(x)}{2}$ on Goods 2 and 3. These shares are of equal size as the heterogeneity of individual household consumption averages out in the aggregate. The entropy of spending on the aggregate level when the spending of each of the two households is equal to x is therefore given by

$$\hat{E}(x) = -\hat{s}_1\ln\hat{s}_1 - \hat{s}_2\ln\hat{s}_2 - \hat{s}_3\ln\hat{s}_3 = -\hat{s}_1\ln\hat{s}_1 - 2\hat{s}_2\ln\hat{s}_2 \tag{12}$$

Lemma 1. *Suppose that $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ (**Condition B**) holds, implying that the spending shares on the basic need Good 1 fall in income x (i.e. that $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{\partial(\hat{s}_1(x))}{\partial x} < 0$ holds). Then, the entropy of spending on the aggregate level $\hat{E}$ continuously rises in x when $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ holds (Case i), while it first rises in x (for $\underline{x} \leq x < \check{x}$) and then falls in x (for $\check{x} < x < \infty$) when $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$ holds and when γ_1 is sufficiently large (Case ii).*
*(In Case ii, γ_1 is sufficiently large if $\gamma_1 > p\gamma_2$ and $\gamma_2 \geq 0$ (**Condition C1**) or if $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{B2}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{B2}\right)}$ and $\gamma_2 < 0$ hold (**Condition C2**)).*

Proof. See Appendix A1. □

The parameter conditions in this Lemma guarantee that poor households (for which x is close to $\underline{x}$) spend more than one third of their budget on the basic need Good

1 and that the budget share of this Good falls as income grows, implying that the shares $\hat{s}_2(x) = \hat{s}_3(x)$ rise in x. At low levels of income, an increase in income therefore always leads to a rise in the entropy of spending on the aggregate level $\hat{E}$. This is due to the fact that it leads to a smoother allocation of consumption spending over the three goods (note that entropy is maximal if one third of the budget is spent on each of the goods). If the budget share of Good 1 still exceeds one third when income becomes infinitely large (Case i), $\hat{E}$ therefore always rises in x. When the budget share of Good 1 falls below one third at a finite income threshold $\check{x} > \underline{x}$, there is an inverse-U relation between $\hat{E}$ and x. $\hat{E}$ then first rises in x, but falls in x once $x > \check{x}$ holds. For the described parameter values, the model can therefore generate **Stylized Fact 2** concerning the shape of the Engel curve for spending diversity on the group level.

The primary purpose of the model is not to exactly fit the data in the case of three goods, but to rather provide qualitative insights that can also be applied to the case with more than three goods. At the same time, the assumptions about the shares of the aggregated expenditures $\hat{s}_j$ made in Lemma 1 do indeed match the data quite well in the case of three goods. Table 2 shows that in this case, the average budget share of food (partitioning the population into income deciles) exceeds one third for all but the richest income decile and that it falls as income rises (Engel's Law). Moreover, the average budget shares for goods and services lie below one third for low income levels and tend to rise in income.[21] Figure 3 (the top right figure) shows that the entropy of spending on the aggregate level $\hat{E}$ rises in income for low and intermediate income levels and that it (slightly) falls in income for high income levels. This pattern is therefore in line with Lemma 1.

When $x > \underline{x}$ holds, the entropy E_i of individual household consumption spending is lower than the entropy of spending on the aggregate level (i.e. $E_i < \hat{E}$ holds). This is due to the heterogeneity of preferences as the budget shares are more unequal at the individual level, thereby implying a lower entropy (and therefore consumption diversity) at this level.[22] This is in line with **Stylized Fact 3** that the Engel curve for spending diversity on the aggregate level is situated above the Engel curve for

[21] However, unlike in the stylized modelling example, these shares are not of equal size and tend to be larger for goods than for services. By assuming that the subsistence consumption level for food (γ_2) exceeds that for services (γ_3), the modelling example could be extended to also account for this feature.

[22] Given that $s_{B2}(x) > s_{B3}(x)$ and $s_{B2}(x) + s_{B3}(x) = 2\hat{s}_2(x)$, the term $-s_{B2}(x)\ln s_{B2}(x) - s_{B3}(x)\ln s_{B3}(x)$ falls in $s_{B2}(x)$ and is therefore maximal if $s_{B2}(x) = s_{B3}(x)$ holds. This implies that E_i is maximal and that $E_i = \hat{E}$ holds if $s_{B2}(x) = s_{B3}(x)$.

spending diversity on the individual level. The following proposition analyzes the relation between E_i and $\hat{E}$:

Proposition 1. *Suppose that* $\gamma_1 > -2\gamma_2 p$ ***(Condition D)*** *and that the conditions from Lemma 1 (leading to either Case i or ii) are satisfied, implying that* $\frac{\partial(s_{i1}(x))}{\partial x} < 0$, $\frac{\partial(s_{B2}(x))}{\partial x} > 0$ *and that* $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ *hold. Then, the (non-negative) difference* $\hat{E} - E_i$ *between the entropy of spending on the individual household level and the entropy of spending on the aggregate level increases in income* x *if* $\gamma_2 > 0$*, while it first decreases and then increases in income when* $\gamma_2 < 0$*.*
Formally, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ *when* $\gamma_2 > 0$*, while* $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ *for* $\underline{x} \leq x < \tilde{x}$ *and* $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ *for* $\tilde{x} < x < \infty$ *when* $\gamma_2 < 0$ *(when* $\gamma_2 = 0$, $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ $(= 0)$ *holds for* $x > \underline{x}$ $(x = \underline{x})$*).*

Proof. See Appendix A2. □

This proposition shows under which conditions the model can generate **Stylized Fact 4**. The commonly observed empirical pattern of a U-shaped relation between the entropy difference $\hat{E} - E_i$ and x therefore arises when γ_2 is negative, i.e. in the case where the income elasticity of Goods 2 and 3 is sufficiently large. The rarely observed pattern that $\hat{E} - E_i$ continuously rises in x arises if γ_2 is positive, i.e. when the income elasticity of Goods 2 and 3 is relatively low. In the following, both cases are discussed separately, starting with the case where γ_2 is positive (as it is simpler and provides a basis for understanding the other case).

When $\gamma_2 \geq 0$, $E_i(x) = \hat{E}(x)$ holds at the minimal income level $x = \underline{x}$ as all households then consume the same quantities $q_{ij} = \gamma_j$ (see Figure 16. **Figure 17** shows a graphical representation of the Engel curves for spending diversity). When income exceeds the level $\underline{x}$, individual households allocate their spending more unevenly across Goods 2 and 3 than households do on average. $\hat{E}$ then exceeds E_i, and the more so the more heterogeneous individual household tastes are, i.e. the more $\beta_{B2} = \beta_{J3}$ exceed the value $\frac{\bar{\beta}}{2}$ of the average household. As the consumption of individual households becomes more specialized when income rises, $\hat{E} - E_i$ then continuously rises in x. Differences in spending patterns between different households therefore rise when household income grows.

Due to the fact that the consumption patterns of individual households closely reflect average consumption patterns at low levels of income, the assumptions (from Lemma

1) that guarantee that $\frac{\partial \hat{E}}{\partial x} > 0$ holds for low income levels also guarantee that E_i rises in x when x is low. E_i can, however, fall as x rises when x is sufficiently high and when the share of the budget that a household allocates to either Good 2 or 3 becomes disproportionately large. Therefore, a inverse-U relationship between individual consumption entropy E_i and x can exist as observed in **Stylized fact 1**.

Given that the model generates Stylized Fact 1, the finding that $\hat{E} - E_i$ rises in x when $\gamma_2 > 0$ holds implies that there can be the following relations between $\hat{E}$ and x in this case: $\hat{E}$ either continuously rises in x (see Figure 17 on the left hand side), or there is also an inverse-U relation between $\hat{E}$ and x and $\hat{E}$ reaches its maximum for larger values of x than E_i does (see Figure 17 on the right hand side).

When $\gamma_2 < 0$ holds, $\hat{E} > E_i$ holds even at the minimal income level $x = \underline{x}$, as individual households then do not purchase any units of either Good 2 or 3 (i.e. as $s_{B3} = s_{J2} = 0$ holds), while the aggregate spending shares $\hat{s}_2 = \hat{s}_3$ are positive for these goods (a graphical representation is given in **Figure 18**). As $\frac{\partial E_i}{\partial x} = -\frac{\partial s_{i1}}{\partial x}(\ln s_{i1} + 1) - \frac{\partial s_{i2}}{\partial x}(\ln s_{i2} + 1) - \frac{\partial s_{i3}}{\partial x}(\ln s_{i3} + 1)$ and as $\frac{\partial s_{i2}}{\partial x}$ and $\frac{\partial s_{i3}}{\partial x}$ are positive in the case considered in Proposition 1 when $\gamma_2 < 0$ holds (this is shown at the beginning of the proof of Proposition 1) the derivative $\frac{\partial E_i}{\partial x}$ gets infinitely large when s_{B3} or s_{J2} go to zero. This implies that the spending diversity of a household increases substantially when it starts consuming positive quantities of a good that it has not consumed before at lower levels of income. When x is close to $\underline{x}$, $\hat{E} - E_i$ therefore falls in x when $\gamma_2 < 0$ holds as $\frac{\partial E_i}{\partial x}$ exceeds the value of $\frac{\partial \hat{E}}{\partial x}$ which is finite even at the point where $x = \underline{x}$.[23]

There are two underlying reasons why we observe that entropy differences fall in income for low income levels in our data: 1): the variety of goods that a household consumes rises in income when income is not too large (see Figures 1 and 2). As this implies that many of a household's consumption shares rise from zero to positive levels when income rises, the entropy E_i of household consumption rises strongly in income when income is relatively low. 2): As the heterogeneity of preferences implies that households do not necessarily add the same goods to their consumption baskets when their income rises, the variety of goods consumed by individual households lies below that consumed by representative households with the same income (these issues are discussed in more detail in Section 4.1). At the aggregate level, there are consequently less "consumption zeros" and fewer aggregate consumption shares ($\hat{s}_j(x)$) rise from zero to positive levels than individual consumption shares ($s_{ij}(x)$) when income grows. Consequently, $\hat{E}$

[23]This argument can be generalized to the case where there exist more than three consumption goods.

rises less strongly in income than E_i for low income levels and $\hat{E} - E_i$ falls.

Our model can therefore provide some insights for situations in which households do not consume all available goods. However, it's main focus is on the case where households consume positive quantities of all goods, i.e. in which the variety of goods consumed does not rise in income. As Figures 1 and 2 show that the variety of goods consumed does not change much in income once income is sufficiently large, the model is best suited for analyzing such income ranges.

Let us now consider the case where income is so large that all consumption shares are sufficiently distinct from zero in the modelling example with $\gamma_2 < 0$ presented above. Then, the mechanisms that are already at work in the case where $\gamma_2 \geq 0$ become dominant again, and a further increase in income induces households to devote an ever increasing share of their budget towards their preferred consumption good. This reduces the entropy of spending on the individual household level relative to the entropy of spending on the aggregate level as consumption heterogeneity averages out in the aggregate. Consequently, $\hat{E} - E_i$ again rises in x when x is sufficiently large (i.e. when $\tilde{x} < x < \infty$) and increasing spending diversity at the aggregate level can again go along with declining diversity at the individual household level.[24] Given that parameters are such that there is an inverse-U relation between E_i and x (Stylized Fact 1), the fact that there is a U-shaped relation between the entropy difference $\hat{E} - E_i$ and x when $\gamma_2 < 0$ holds therefore implies that the relation between $\hat{E}$ and x can again be of two forms in this case: $\hat{E}$ either continuously rises in x (see Figure 18 on the left), or there is an inverse-U relation between $\hat{E}$ and x and the inverse-U of $\hat{E}$ reaches its maximal level at a higher level of income than the inverse-U of E_i (see Figure 18 on the right).

4 The value of product variety

The insights about the heterogeneity of household demand obtained in the last sections can have important welfare consequences. In this section we discuss these by analyzing the value of product variety. Being able to determine this value is crucial when it comes to designing optimal innovation, trade, and antitrust policies as these policies often

[24] Condition D, which can only be binding if $\gamma_2 < 0$ holds, is imposed to ensure that $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ holds. If this condition is violated, $\frac{\partial\left(\hat{E}(x) - E_i(x)\right)}{\partial x} < 0$ holds for all values of x (this is shown in the proof of Proposition 1). As this case is not in line with the empirical observations, Condition D is imposed in Proposition 1.

involve trade-offs between the variety of goods that are available and other variables (like R&D costs). The analysis is carried out within the three-good example from Subsection 3.1.

In order to analyze the effects of a change in product variety, the situation in which only the "basic need" Good 1 exists is compared to the situation where also Goods 2 and 3 can be purchased, for example because they are introduced through innovation or made accessible through a free trade agreement. While Good 1 is always sold at price $p_1 = 1$, Goods 2 and 3 are now only sold at price $p_2 = p_3 = p$ when they are available, but have an infinite price (or a price that is large enough to reduce demand to zero) when not[25]. In order to to compare the welfare levels of household with and without Goods 2 and 3, the case is considered in which $\gamma_2 = \gamma_3 \leq 0$ holds, i.e. in which there is no required positive subsistence consumption level for Goods 2 and 3.

Again, the case in which $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{B2} = \beta_{J3} > \frac{\bar{\beta}}{2}$ and $\beta_{B3} = \beta_{J2} = \bar{\beta} - \beta_{B2} \geq 0$ is considered in which household Brown prefers Good 2 over Good 3 and household Jones has exactly the opposite preferences and prefers Good 3 over Good 2. As explained above, the aggregated (market) demand for each good does in this case not depend on the extent of preference heterogeneity (i.e. on β_{B2}) and can also be derived from the utility maximization problem of two households with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$) and equal incomes x. Put differently, for any relative price p and total income, the market demand for each good can also be derived from a model with representative households as long as the size of the population and the distribution of income are kept fixed and as long as $x_i \geq \underline{x}$ holds.[26] We can therefore analyze whether a household with heterogeneous preferences ($\beta_{i2} \neq \frac{\bar{\beta}}{2}$) values an increase in product variety in a different way than a representative household with average preferences but the same income does.

This is an interesting question as it allows us to evaluate whether and how ignoring the preference heterogeneity that we have identified as the driving force behind our

[25] For simplicity, we assume prices to be exogenous. We therefore do not consider the possibility that an increase in product variety might affect welfare by changing the price of good 1 when markets are not perfectly competitive.

[26] It should be noted that there does not exist a representative household in the more narrow sense in our setting, as the demand stemming from such a "positive" representative household needs to coincide with the market demand for any distribution of income. In our setting, however, the heteroscedasticity of Engel curves implies that the distribution of income across households with different preferences (i.e. different slopes of Engel curves) does impact the market demand for goods. Furthermore, the fact that expenditure shares depend on income implies that there are non-homotheticicities due to which the market demand for a good depends on both total expenditures and on the size of the population in our setting. In order to obtain the same market demand with and without consumer heterogeneity, we consequently consider the case where both the size of the population and the distribution of income remain fixed.

empirical observations and instead focusing on a simpler model with hypothetical representative households leads to biased welfare results. It should be noted that such a simpler model would also allow us to correctly determine the incentives to innovate in an environment with endogenous innovation by profit seeking firms (at least in a symmetric equilibrium in which the inventors of the Goods 2 and 3 charge the same monopoly prices) as these incentives only depend on the total (market) demand for a newly introduced good. While preference heterogeneity neither affects market demand or "consumer surplus" (when derived as an area under the demand curve), nor the incentives to innovate in our setting, it might, however, nevertheless affect the value that households attribute to an increase in product variety.

While it is obvious that a household benefits more from the introduction of a good that it has a strong preference for than from the introduction of a good that it values less, the question considered here is whether a household benefits more or less from the joint introduction of both Goods 2 and 3 when it puts a larger relative welfare weight β_{ij} on one of them, keeping $\beta_{i2} + \beta_{i3} = \bar{\beta}$ and therefore the total quantity of the two goods that it consumes constant.[27] The extent of preference heterogeneity is then increasing in β_{ij} when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds for a good $j \epsilon \{2; 3\}$.

The value of variety F_i is defined as the increase in income (the "compensating variation" according to Hicks (1942)) that would be required to make a household i with income x_i who can purchase all three goods equally well off if it could only purchase good 1. Put differently, F_i measures the compensation that a household requires to give up on product variety.[28] The value that a household with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$) attributes to variety is denoted by F_a (so that $F_i|_{\beta_{ij}=\frac{\bar{\beta}}{2}} = F_a$ holds). The case is considered in which $x_i > \underline{x}$ holds so that households are rich enough to consume positive quantities of all three goods if they are available.

Proposition 2. *An interior solution for F_i (and F_a) exists if either $\varepsilon > 1$ and $x_i > \underline{x}$, or if $\varepsilon < 1$ and $\underline{x} < x_i < \hat{x}$ (with $\underline{x} = \gamma_1 + 2p\gamma_2 - \frac{(1-\bar{\beta})\gamma_2 p^{\varepsilon} + \bar{\beta}p\gamma_2}{\bar{\beta} - \beta_{i2}}$ and $\hat{x} \equiv \gamma_1 + 2p\gamma_2 +$*

[27]By looking at the joint introduction of two goods, one does not need to consider individual risk preferences that might play a role when instead the welfare consequences of the introduction of only one good of ex ante unknown desirability were studied.

[28]Seen from a different angle, the value of product variety F_i is defined as the increase in income (the "equivalent variation" according to Hicks (1942)) that is required to make a household i with income x_i that can only purchase good 1 equally well off as being able to purchase all three goods (with income x_i) would. Alternatively, F_i could be defined as the income that a household with income x_i that can only consume Good 1 is maximally willing to give up (i.e. the negative compensating variation) in order to be able to purchase all three goods. We do not use this definition here as it leads to more difficult calculations.

$$\left[\frac{(\beta_{i2})^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}+\left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ when } \beta_{i2} \geq \frac{\bar{\beta}}{2}).$$

a) Given that $\gamma_2 = \gamma_3 < 0$ and that an interior solution exists, a household i with heterogeneous preferences ($\beta_{ij} \neq \frac{\bar{\beta}}{2}$ for $j \epsilon \{2;3\}$, but $\beta_{i2} + \beta_{i3} = \bar{\beta}$) values variety more than a household with average preferences ($\beta_{a2} = \beta_{a3} = \frac{\bar{\beta}}{2}$) does and the more so, the more heterogeneous these preferences are (formally, $F_i > F_a$ holds, with $\frac{\partial F_i}{\partial \beta_{ij}} > 0$ when $\beta_{ij} > \frac{\bar{\beta}}{2}$ is satisfied for a good $j \epsilon \{2;3\}$, and $\frac{\partial F_i}{\partial \beta_{ij}} = 0$ when $\beta_{i2} = \beta_{i3} = \frac{\bar{\beta}}{2}$).

When $\varepsilon < 1$ (complementary goods) and when x_i approaches its upper bound $\hat{x}$, F_i goes to infinity while F_a remains finite, implying that households with heterogeneous preferences value variety infinitely more than households with average preferences do (When instead $\varepsilon > 1$ (substitutable goods), $\lim\limits_{x_i \to \infty} \frac{F_i}{F_a} = 1$ holds, so that the values of variety become the same in this limit case).

b) When $\gamma_2 = \gamma_3 = 0$ and when an interior solution exists (requiring $\varepsilon > 1$ in this case), a household i with heterogeneous preferences values variety the same as a household with average preferences does (so that $F_i = F_a$).

c) When the income elasticity of Goods 2 and 3 increases (i.e. when these goods become more luxurious) because γ_2 becomes more negative, the value of variety decreases (formally, $\frac{\partial F_i}{\partial \gamma_2} > 0$ and $\frac{\partial F_a}{\partial \gamma_2} > 0$).

d) When income x_i increases, the value of variety increases, and the more so the more heterogeneous preferences are (formally, $\frac{\partial F_i}{\partial x_i} > \frac{\partial F_a}{\partial x_i} > 0$, and $\frac{\partial^2 F_i}{\partial \beta_{i2} \partial x_i} = \frac{\partial^2 F_i}{\partial x_i \partial \beta_{i2}} > 0$ when $\beta_{i2} \neq \frac{\bar{\beta}}{2}$).

Proof. See Appendix A3. □

Even though aggregated (market) consumption can be derived from the utility maximization problem of representative households with average preferences, such hypothetical households therefore value variety less than households with heterogeneous preferences (and equal incomes) do when $\gamma_2 < 0$ holds.[29] Under this parameter condition, newly introduced goods have a relatively high income elasticity. This is is a highly relevant case when it comes to various applications in the areas of innovation and trade. Moreover, the observation that there is in many cases a U-shaped relation between the entropy difference $\hat{E} - E_i$ and x in our data can be explained by our model

[29]When all three goods are available, all households are, however, equally affected when the price p of Goods 2 and 3 changes, as the utility of consuming positive quantities of all goods is independent of β_{i2} (see Appendix A3).

when $\gamma_2 < 0$ holds. Therefore, this case also seems to be relevant for the sample of goods that we look at in our empirical study.

Interestingly, F_i/F_a becomes infinitely large when good are complementary and when x reaches its upper bound, as F_i reaches infinity while F_a remains finite in this case. The reason for this is the following: when x rises, the utility of only consuming Good 1 remains bounded from above in this case, while there is no such saturation in the case when there are three goods. As the saturation effect is stronger for households with heterogeneous preferences, no amount of additional income can compensate them for the loss of product variety when x is sufficiently large, while representative households with the same x are still willing to give up on product variety for a finite compensation. Studying the welfare of representative households without taking the empirically observed heterogeneity into account can consequently lead to an enormous underestimation of the true value that households attach to product variety. This is the main result of our theoretical analysis.[30]

In the knife-edge case where $\gamma_2 = \gamma_3 = 0$, so that there are no (negative) subsistence consumption levels for the newly introduced goods, preference heterogeneity has no effect on the value of product variety. This implies that preference heterogeneity does not matter for the value of product variety when preferences are, as commonly assumed, homothetic, i.e. when $\gamma_1 = \gamma_2 = \gamma_3 = 0$ holds. Such homothetic preferences, however, are not only inconsistent with our empirical finding of a predominantly U-shaped relation between entropy-differences and income, but also imply that the consumption shares of different goods are independent of household income. This is clearly at odds with empirical evidence, including our results presented in Table 2, in which it can be seen that the consumption shares of food fall in income, while those of goods and services rise in income.

The result that an increase in the heterogeneity of preferences increases the value of product variety when $\gamma_2 < 0$ holds is driven by the following mechanism: the utility that a household with a given income obtains when it consumes positive quantities

[30] Jerison (2016) shows that large welfare differences between individual and representative households can result if there is a Giffen good for the representative consumer and for some actual consumer. This is not the driving force behind our results as there are no Giffen goods in our setting: it can be shown that the optimal quantities $q*_{i2}$ and $q*_{i3}$ fall in p when $x > \gamma_1 - \frac{2\left(1-\bar{\beta}\right)p^{\varepsilon}(1-\varepsilon)\gamma_2}{\bar{\beta}+\varepsilon\left(1-\bar{\beta}\right)p^{\varepsilon-1}} \equiv \breve{x}$ holds. As it turns out that $\breve{x} < \underline{x}$ for $\beta_{i2} \geq \frac{\bar{\beta}}{2}$, this inequality is indeed satisfied for the parameter range that we consider, implying that $q*_{i2}$ and $q*_{i3}$ (and also the optimal quantities of consumers with average preferences) fall in p. It can, moreover, be shown that the optimal quantities $q*_{i1}$ consumed of Good 1 would also fall if its price (which is normalized to 1 here) would rise. Consequently, there are no Giffen goods in our setting.

of all three goods turns out to be independent of the individual values of β_{i2} and β_{i3}, as long as $\beta_{i2}+\beta_{i3}=\bar{\beta}$ holds. Contrary to that, the utility of a household who only consumes Good 1 falls in β_{ij} when $\beta_{ij}>\frac{\bar{\beta}}{2}$ holds for $j\epsilon\{2;3\}$ as such an increase in preference heterogeneity reduces the utility derived from the negative subsistence consumption levels $\gamma_2=\gamma_3$.[31] Consequently, the utility gain derived from being able to purchase all three goods (in optimal quantities) instead of only consuming Good 1 increases in β_{ij} if $\beta_{ij}>\frac{\bar{\beta}}{2}$ holds, implying a larger value of product variety.[32]

The result that a household values the joint introduction of two goods more when its preferences regarding these goods are more heterogeneous can be generalized. It also holds in settings in which, when these goods are not available, households not only have access to a single good (Good 1), but to several goods of each of which they consume positive quantities (see Appendix A3).

Part c) of proposition 2 shows that the value of product variety falls when newly introduced goods become more luxurious and when their income elasticity falls due to a more negative value of γ_2. This result is quite intuitive as, holding income constant, households value more luxurious goods less than goods that satisfy more basic needs.

Moreover, it is shown in part d) that an increase in income increases the value of product variety, i.e. that richer households need a larger compensation for accepting a reduction in the set of goods that they can consume. This (absolute) increase in F_i induced by an increase in x_i is larger for households with more heterogeneous preferences.[33]

As can be seen in Appendix A3, the value of variety F_i and also the effect of preference heterogeneity (β_{i2}) on F_i depend on many different parameters: on income (x_i), the luxuriousness of the newly introduced goods ($-\gamma_2$), preference heterogeneity (β_{i2}), and also the substitutability between goods (ε), the subsistence consumption level of Good

[31] The utility of a household who only consumes Good 1 is given by $U_i(1) = \left(\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}\left(x_i-\gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}}+\beta_{i2}^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}+\left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$

[32] It should be noted that this result is derived for values of x_i for which households always consume positive quantities of all available goods. For lower values of x_i, it is possible that a household with heterogeneous preferences consumes positive quantities of one of the newly introduced Goods (2 and 3) and has a positive value of variety ($F_i>0$), while a household with average preferences does not consume any of these goods, implying that it does not value variety at all ($F_a=0$). We do not consider this case as aggregated demand is not independent of preference heterogeneity for such parameter constellations. This implies that the aggregated demand for each good cannot be derived from a model with representative households that all have the same average preferences.

[33] As households with more heterogeneous preferences also have a larger initial value of F_i, it would therefore be interesting to analyze how the relative value of variety F_i/F_a depends on income. We, unfortunately, did not succeed to carry out such an analysis in full and could only show that $F_i/F_a>1$ holds and derive the limit cases that are presented in Proposition 2a.

1 (γ_1), the relative consumption importance of Goods 2 and 3 relative to Good 1 ($\bar{\beta}$) and the price of Goods 2 and 3 relative to the price of Good 1 (p). Consequently, F_i can in practice be very different for different goods, and also the effect of preference heterogeneity on F_i can vary considerably across goods. It is therefore difficult to come up with generalizable quantifications of the importance of preference heterogeneity for the value of product variety.

4.1 Accounting for variety demand

The above analysis focused on the case where $x_i > \underline{x}$ holds, implying that households are rich enough to purchase positive quantities of all available goods. Moreover, it analyzed the limit case where $x_i = \underline{x}$ holds in order to get some insights about what happens when some consumption shares become zero. In order to properly account for the empirically observed pattern that the variety of goods consumed by a household tends to rise in household income (see Figures 1 and 2 and previous studies by Jackson (1984) and Falkinger and Zweimüller (1996)), the case in which $x_i < \underline{x}$ holds has to be considered.

When $x_i < \underline{x}$ and when $\gamma_j > 0$ holds for some goods and $\gamma_j < 0$ for others, all households purchase positive quantities of the goods for which $\gamma_j > 0$ holds, while only households with sufficient income purchase positive quantities of goods for which $\gamma_j < 0$ holds (as the marginal utility of the first unit of such goods is finite while that of goods with $\gamma_j > 0$ is infinite). The variety of goods consumed therefore increases in income x_i when there are several goods for which $\gamma_j < 0$ holds. Even when the parameters γ_j are the same for all households, households that differ with respect to the parameters β_{ij} might then increase the variety of goods that they consume in a different order. This becomes clear by looking at the example with three goods and two households (Brown and Jones) from Section 3.1, in which it is assumed that $\gamma_2 < 0$, $\beta_{i1} = 1 - \bar{\beta}$, $\beta_{B2} = \beta_{J3} > \frac{\bar{\beta}}{2}$ and $\beta_{B3} = \beta_{J2} = \bar{\beta} - \beta_{B2} \geq 0$: In this case, $\underline{x} = \gamma_1 + 2p\gamma_2 - \gamma_2 \left[\frac{(1-\bar{\beta})p^{\varepsilon} + \bar{\beta}p}{\bar{\beta} - \beta_{B2}} \right]$ holds (see the proof of Lemma 1), implying that household Brown (Jones) stops consuming Good 3 (2) when income falls below the level $\underline{x}$. Applying equation 6 to the case where households only purchase the two remaining goods, it can be shown that households stop consuming two goods and spend all their income on the basic need Good 1 when there is a further fall in x_i below the threshold $\dot{x} \equiv \gamma_1 - \gamma_2 \frac{1-\bar{\beta}}{\beta_{B2}} p^{\varepsilon} < \underline{x}$. When incomes increase from a level $x_i < \dot{x}$

to a level $x_i > \underline{x}$, households therefore expand the variety of goods that they consume from one to three, but in a different order: while household Brown purchases Goods 1 and 2 when income lies in the range $\dot{x} < x_i < \underline{x}$, household Jones purchases Goods 1 and 3 in this range as tastes are heterogeneous with respect to Goods 2 and 3.

Applying these insights to a more general setting with many goods j, the direction in which variety demand grows can then vary across the population when households differ with respect to the parameters β_{ij}. At low income levels, households with different preferences then not only purchase different quantities of goods, but the set of goods they consume may also vary across households. This implies that the average consumption basket for the group consists of a larger variety of goods than the individual consumption baskets for each household belonging to that group. When individual goods are grouped into broader consumption categories, different households then, moreover, pick the goods which they consume in a more uneven way from these categories than households do on average. This implies that the "diversity of the variety demand" of a household with income x_i is lower than the diversity of the average consumption basket of all households with income x.

This is indeed the case when we look at the data: Figure 14 presents the diversity of variety demand across 12 expenditure categories at the household level and representative household level using data from the year 2000. This figure is derived in the following way: goods are grouped into 12 broader categories indexed by h, with the total number of goods in category h given by N_h. Denoting the number of different goods (i.e. the varieties) that household i consumes within category h by n_{ih}, we then determine the fractions $\frac{n_{ih}}{N_h}$ for all households and categories. Normalizing by $\sum_{h=1}^{12} \frac{n_{ih}}{N_h}$ then gives the (relative) "variety shares" $\theta_{ih} \equiv \frac{\frac{n_{ih}}{N_h}}{\sum_{h=1}^{12} \frac{n_{ih}}{N_h}}$ that add up to one. The entropy measure described in Section 2 is then applied to these variety shares in order to estimate the diversity of household variety demand

$$D_i = \sum_{h=1}^{12} -\theta_{ih} ln \theta_{ih}$$

across the 12 expenditure categories.

For the representative households, average variety demand is calculated as

$$\hat{D} = \sum_{h=1}^{12} -\hat{\theta}_{dh} ln \hat{\theta}_{dh}$$

for the same year. In order to determine $\hat{D}$, households are grouped into deciles and the individual varieties n_{ih} are replaced by the variety $\hat{n}_{dh}$ of goods of category h consumed by decile d (i.e. by the number of all goods of which positive quantities are consumed by at least one household falling into the decile). The entropy is therefore derived for the variety shares at the group level which are given by $\hat{\theta}_{dh} \equiv \frac{\frac{\hat{n}_{dh}}{N_h}}{\sum_{h=1}^{12} \frac{\hat{n}_{dh}}{N_h}}$. Figure 14 shows that the diversity of variety demand at the household level D_i is lower than the diversity of variety demand $\hat{D}$ at the representative (decile) level. This implies that individual households with a certain income do not all consume the same varieties and that they pick the varieties that they consume more unevenly from the different consumption categories than households do on average. This is in line with the predictions of the theoretical analysis from above. Figure 14 moreover shows that both D_i and $\hat{D}$ rise in income x. When income rises, households therefore not only consume a larger variety of goods, but also pick the varieties that they consume in a more even way from the different expenditure categories, so that their consumption baskets become more diverse. Figure 15 presents the estimated difference between the diversity of variety demand on the household and the representative level and shows that this difference falls in income. When households become richer, their pattern of variety consumption therefore become more similar to those at the aggregate level. One reason for this might be that there are less non-consumed varieties left for richer households (who already consume more varieties), limiting the scope to increase variety demand in heterogeneous ways.

Figures 1 and 2 show that the variety of goods consumed is fairly stable in income once income lies above a certain threshold (of around 200 pounds per week). The same holds true for the diversity of variety demand curves. Consequently, changes in variety consumption seem to be less of a driver of changes in consumption heterogeneity at larger income levels. Instead, our finding that spending diversity at the household level falls in income for high income levels while spending diversity at the group level tends to rise (or to fall less) in income seems to be driven by changes in the quantities of different goods that are consumed (and not by changed in the variety of goods of which positive quantities are consumed). Therefore, the assumption made in Section 3.1 that households consume positive quantities of all goods does not seem inappropriate.

5 Conclusion

As noted in James Heckman's 2001 Nobel Memorial Lecture, one of the most important discoveries emerging from micro-econometric research was evidence of the pervasiveness of heterogeneity and diversity in economic life (Heckman, 2001).

In this paper, we have analyzed the heterogeneity of household spending patterns across different levels of aggregation. Specifically, we have studied how the diversity of spending depends on household income. At the level of individual households, we have shown that there exists an inverse-U relationship between spending diversity and income. This means that relatively poor households allocate their spending more evenly across different goods when their income rises, while relatively rich households tend to concentrate their spending on particular goods when their income rises, so that there exists a saturation (and decline) of spending diversity in the latter case.

In contrast, different patterns emerge when we examine the diversity of spending on a more aggregate level by studying groups of households with similar income levels. We find that this diversity of spending on the aggregate level either always rises in income or that it only starts to fall at income levels that are higher than the income levels at which individual household spending diversity starts to fall. There is therefore an income range for which individual household spending diversity falls in income while spending diversity at the aggregate level paradoxically rises in income. We explain this pattern by the fact that individual households concentrate their spending on different goods when their income rises, implying that aggregation across households masks the decline of spending diversity observed at the individual level. The pattern at the aggregate level is therefore emergent in the sense that it does not reflect the pattern found at the individual level.

We present a model in which households have heterogeneous non-homothetic preferences that allows to explain the Stylized Facts. We then use this model to assess the pitfalls of making the simplifying assumption that every household has the same (average) preferences, i.e. that there is a representative household. The main result from our theoretical analysis is that ignoring preference heterogeneity and instead analyzing a representative household model leads to a (potentially very large) underestimation of the value of product variety. This result holds even though the representative household model is well suited to explain aggregated consumption data in the sense that aggregated consumption levels derived from it coincide with those derived from the more general model with heterogeneous households. As our empirical analysis shows that heterogeneity of household consumption patterns and non-homotheticities are a prevalent feature of the data, it therefore indicates that there might be considerable benefits in deviating from the representative household assumption and in explicitly taking preference heterogeneity into account.

It would clearly be interesting to empirically estimate the value of product variety and the effect that preference heterogeneity has on it. Our model shows that this value depends on many parameters and that it can therefore be very different for different goods and in different contexts. We leave a careful estimation of the value that product variety has in relevant real-world examples for future research.

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Tables

Table 1: Categories of the UK Family Expenditure Survey,2000

Category	Examples of spending
Food	Milk, Eggs, vegetables, meats, sweets, non-alcoholic beverages. Take away meals, food bought and consumed at work and school.
Fuel Light and Power	Gas, Electricity, Coal, bottled gas, paraffin, wood.
Alcoholic Drinks	Beer, Lager, Cider, Spirits Liqueurs.
Tobacco	Cigarettes, Pipe tobacco, cigars
Clothing and Footwear	Outerwear, Underwear, Clothing accessories, Footwear, Haberdashery and clothing materials
Household goods	Furniture and Furnishings, Electrical and gas appliances. Hardware, decorative goods. Toilet paper, Pet and garden expenditure.
Domestic and Paid services	Childcare, domestic help, laundry, postage and telephones, subscriptions and stamp duty.
Personal Goods and Services	Hairdressing, cosmetic requisites. Baby goods, medicines and medical goods. Personal effects and travel goods.
Motoring Expenditure	Accessories, parts, repairs and servicing of motor vehicles. Petrol and oil. Insurance, driving lessons and other payment.
Travel	Fares, other transport costs, Purchase and maintenance of non-motor vehicles.
Leisure Goods	TV, video and Audio equipment. Sports, camping and outdoor good and equipment. Newspapers, magazines, books and stationary. Toy, hobbies and photography.
Entertainment and Education Services	Cinema, spectator sports, TV rental and subscription, hotels and holiday expenses, betting stakes, educational fees and maintenance, Ad hoc school expenditure, betting stakes.

Table 2: Average budget shares for food, goods and services per decile

2015				**2010**				**2005**			
Income	Food	Goods	Services	Income	Food	Goods	Services	Income	Food	Goods	Services
30.46	0.66	0.23	0.11	31.01	0.63	0.22	0.15	28.49	0.64	0.21	0.14
58.28	0.58	0.28	0.14	56.46	0.58	0.26	0.16	52.82	0.56	0.28	0.16
78.70	0.54	0.31	0.15	75.07	0.53	0.29	0.18	70.31	0.52	0.30	0.18
98.02	0.51	0.32	0.16	93.36	0.52	0.31	0.17	86.61	0.49	0.33	0.18
118.26	0.48	0.34	0.18	110.87	0.48	0.34	0.18	102.73	0.46	0.36	0.18
139.94	0.48	0.34	0.19	130.83	0.46	0.35	0.19	121.41	0.44	0.36	0.20
166.13	0.45	0.36	0.19	154.27	0.42	0.37	0.21	143.86	0.42	0.38	0.20
197.98	0.40	0.36	0.23	185.21	0.40	0.38	0.22	171.34	0.38	0.39	0.23
247.88	0.39	0.37	0.24	228.88	0.37	0.40	0.22	216.45	0.35	0.40	0.25
370.63	0.30	0.39	0.31	336.60	0.29	0.41	0.30	339.84	0.26	0.42	0.32
2000				**1995**				**1990**			
Income	Food	Goods	Services	Income	Food	Goods	Services	Income	Food	Goods	Services
31.23	0.61	0.21	0.18	25.40	0.64	0.19	0.16	20.49	0.68	0.16	0.16
51.89	0.55	0.26	0.19	39.57	0.58	0.23	0.19	32.93	0.61	0.21	0.18
67.40	0.50	0.31	0.20	50.63	0.55	0.26	0.20	42.49	0.56	0.25	0.19
83.35	0.47	0.34	0.19	62.04	0.52	0.29	0.19	52.30	0.53	0.28	0.19
100.53	0.43	0.39	0.18	73.98	0.48	0.32	0.19	63.37	0.50	0.31	0.19
119.36	0.42	0.41	0.18	86.80	0.46	0.34	0.20	74.80	0.47	0.35	0.18
140.54	0.38	0.44	0.18	101.81	0.43	0.37	0.20	89.49	0.44	0.36	0.20
166.62	0.37	0.44	0.20	121.41	0.42	0.38	0.21	108.55	0.39	0.39	0.22
203.71	0.33	0.47	0.20	150.26	0.38	0.42	0.21	138.44	0.36	0.39	0.25
292.12	0.28	0.48	0.24	219.75	0.31	0.42	0.27	215.15	0.27	0.44	0.29

Note: Each row represents a decile. Income is equal to the average of total weekly household expenditures within the decile. The values for Food, Goods and Services represent the budget share of each category. Consistent with Engel's Law, the budget share for food expenditure tends to decline as income rises.

Table 3

Category	1990		1995		2000		2005		2010		2015	
	$\gamma_i t$	$\delta_i t$	$\gamma_i t$	$\delta_i t$	$\gamma_i t$	$\delta_i t$	$\gamma_i t$	$\delta_i t$	$\gamma_i t$	$\delta_i t$	$\gamma_i t$	$\delta_i t$
Fuel Light and Power	0.00376 ***	0.151***	0.00271*	0.133***	-0.0000976	0.0710**	0.00180**	0.0590***	0.00239*	0.195***	0.00480***	0.396***
	(0.004)	(0.002)	(0.053)	(0.001)	(0.923)	(0.014)	(0.029)	(0.003)	(0.070)	(0.002)	(0.000)	(0.000)
Food	0.166***	3.779***	0.232***	5.182***	0.202***	5.425***	0.194***	9.503***	0.245***	9.219***	0.262***	10.51***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Alcoholoic Drinks	0.0707***	1.545***	0.0946***	2.624***	0.0836***	2.697***	0.0723***	3.234***	0.0847***	3.425***	0.0920***	3.618***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Tobaco	0.0105***	0.228***	0.0135***	0.479***	0.0147***	0.501***	0.00587***	0.306***	0.00896***	0.570***	0.00725***	0.434***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Clothing and Footwear	0.199***	11.73***	0.235***	12.60***	0.186***	11.78***	0.159***	11.98***	0.163***	11.26***	0.149***	9.904***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Household goods	0.244***	31.49***	0.187***	12.95***	0.178***	15.10***	0.178***	39.16***	0.147***	14.68***	0.162***	14.31***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Domestic and Paid services	0.0887***	9.489***	0.110***	11.36***	0.107***	11.06***	0.113***	23.44***	0.104***	14.93***	0.109***	20.08***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Personal Goods and Services	0.0887***	5.007***	0.114***	4.123***	0.136***	9.390***	0.104***	7.669***	0.111***	5.191***	0.116***	8.103***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Motoring Expenditure	0.154***	16.37***	0.149***	10.78***	0.164***	13.03***	0.148***	20.94***	0.162***	10.94***	0.158***	17.77***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Travel	0.0623***	7.667***	0.0492***	5.263***	0.0525***	6.674***	0.0663***	15.47***	0.0566***	6.160***	0.0606***	9.553***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Leisure Goods	0.119***	9.370***	0.129***	7.090***	0.151***	17.21***	0.154***	18.65***	0.175***	24.33***	0.142***	19.95***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Entertainment and Education Services	0.218***	34.18***	0.152***	20.04***	0.164***	27.82***	0.270***	83.48***	0.183***	36.53***	0.207***	42.03***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

*p-values in parentheses, * p<0.1, ** p<0.05, *** p<0.01* *Notes:* $\gamma_i t$ and $\delta_i t$ are coefficients of total income in the following regressions: $Expenditure_{it} = \beta_0 it + \gamma_i t * income_t + \boldsymbol{\beta} * \boldsymbol{X} + \epsilon_{it}$ and $(\epsilon_i t)^2 = \alpha_0 it + \delta_i t * income_t + \boldsymbol{\alpha} * \boldsymbol{X} + \mu_i t$. $\mathbf{X}$ is a vector of demographic variables including: (1) location, (2) type of household, (3) composition of household, (4) number of children, (5) number of workers, (6) age of head of household by range, (7) economic position of head of household, (8) households with married women. α, β are matrices of coefficients. $\epsilon_i t$ is the residual of the first regression. i,t are subscripts for categories and year.

Figures

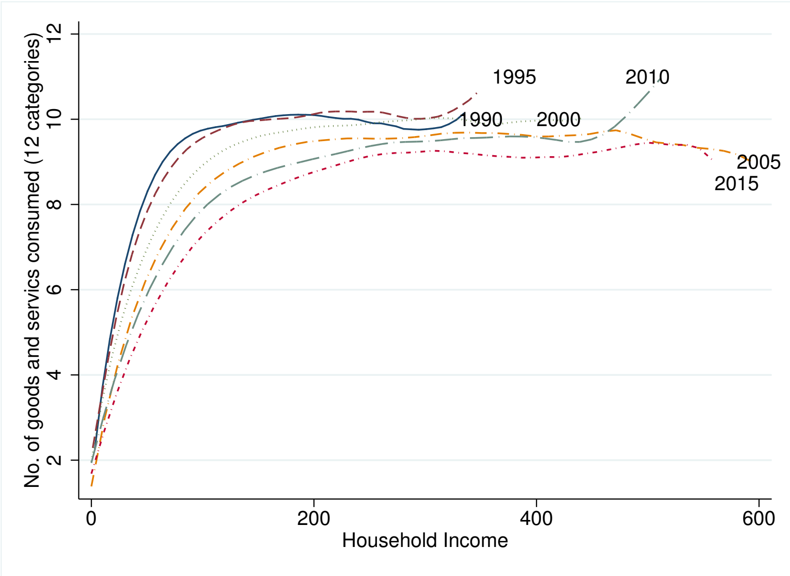

Figure 1: Total variety consumed over income (12 categories)

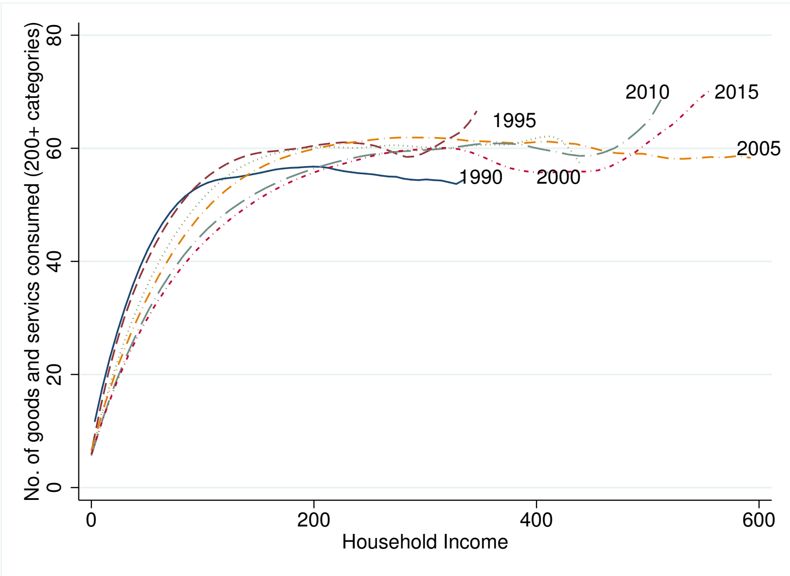

Figure 2: Total variety consumed over income (200+ categories)

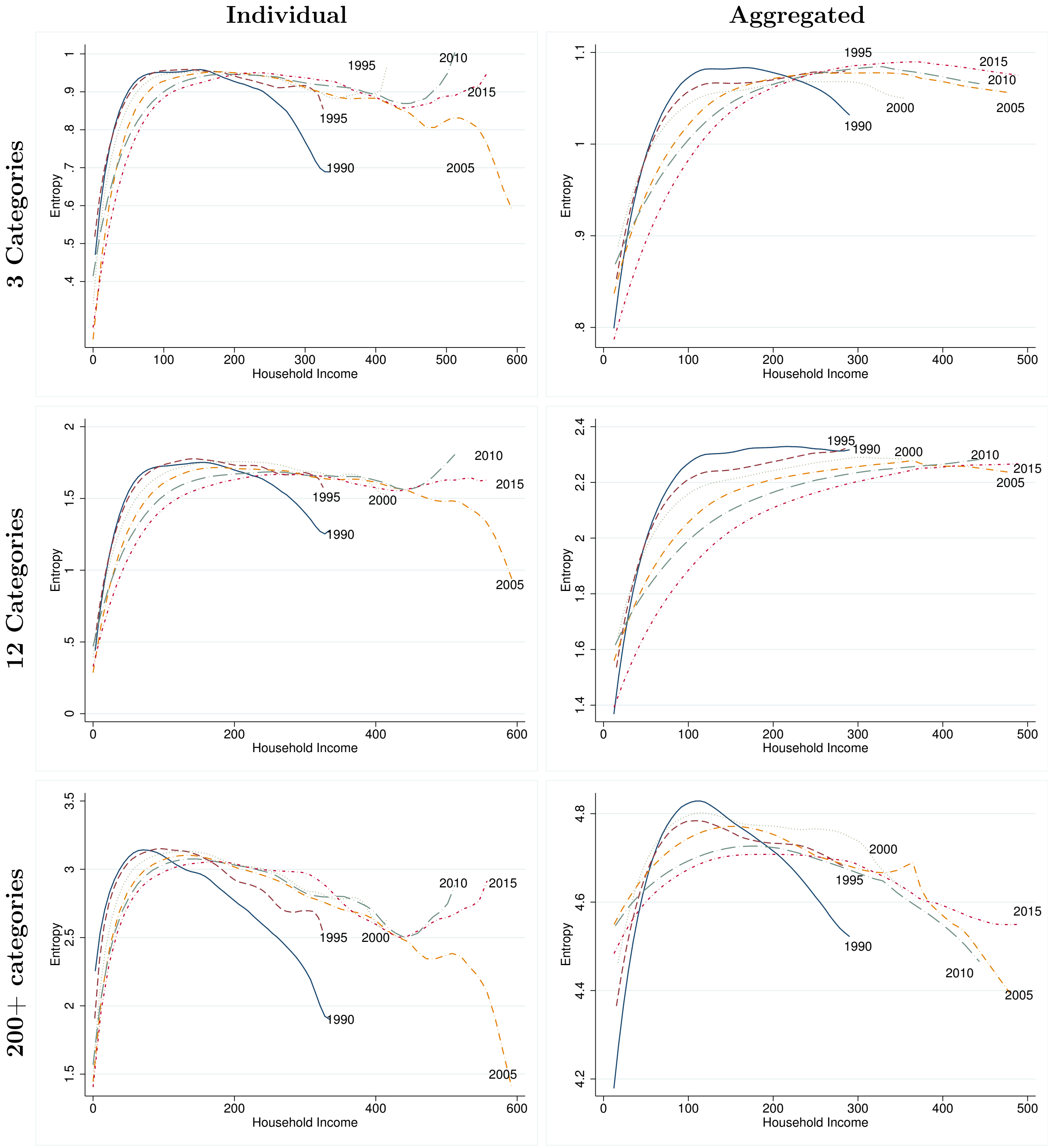

Figure 3: The Engel curves for spending diversity

Notes: The Figures on the left show the entropy E_i of consumption spending of individual households, while the Figures on the right depict the entropy $\hat{E}$ of spending on the aggregate level for groups of households with similar income levels. Households are aggregated into 50 representative groups with similar income levels. Each row represents a different level of aggregation across expenditure categories. In the first row, three broad categories are used: food, goods and services. The middle row uses the 12 expenditure categories listed in Table 1, and the bottom row uses the maximum level of disaggregation of 200+ categories. The number of observations was 5,949 in 1990, 5,881 in 1995, 5,771 in 2000, 5,669 in 2005, 4,530 in 2010 and 4,217 in 2015.

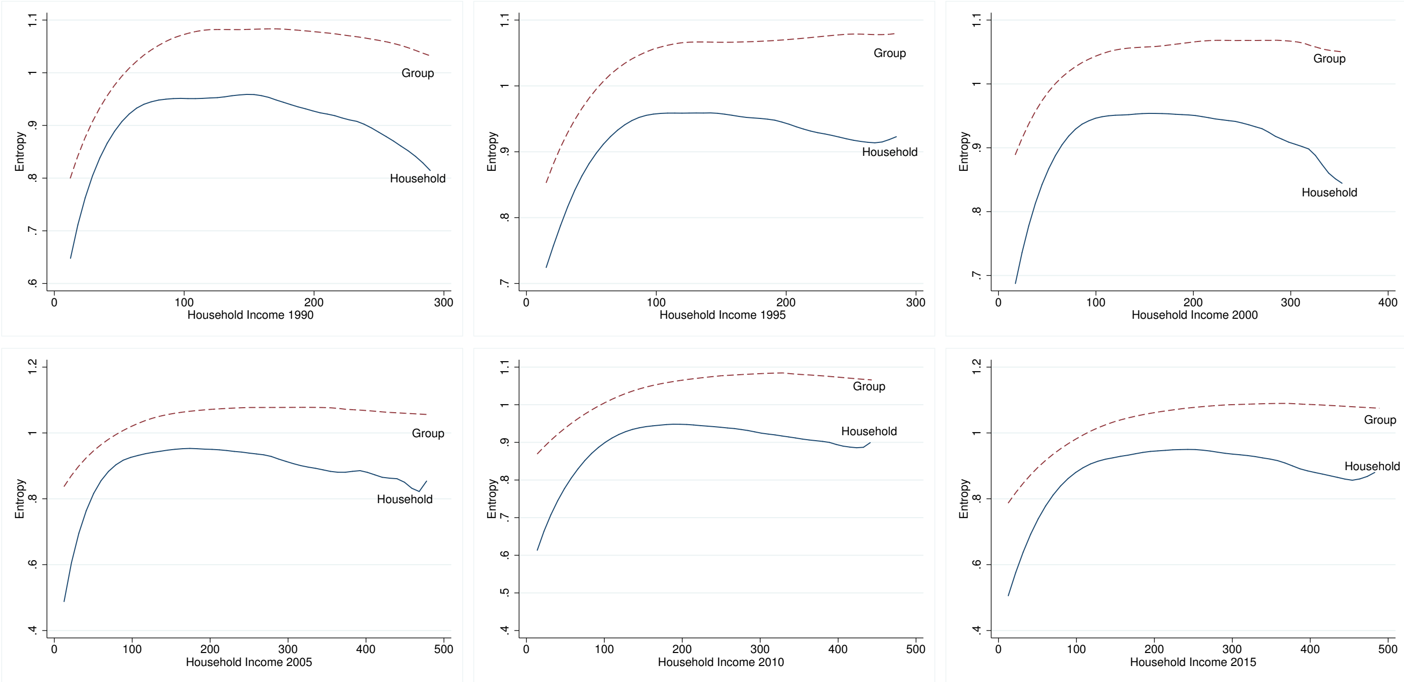

Figure 4: The Engel curves for spending diversity on the household and aggregate (group) level (aggregation level: 3 expenditure categories)

Notes: The Figures depict spending diversity on the household level (solid line, E_i) and on the aggregate level (dashed line, $\hat{E}$) for 1990 (top left), 1995 (top middle), 2000 (top right), 2005 (bottom left), 2010 (bottom middle) and 2015 (bottom right). Expenditure categories consist of 3 categories - food, goods and services. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average income of the poorest and above the average income of the richest income group. As a result, these curves cover a smaller income range than those displayed in Figure 3

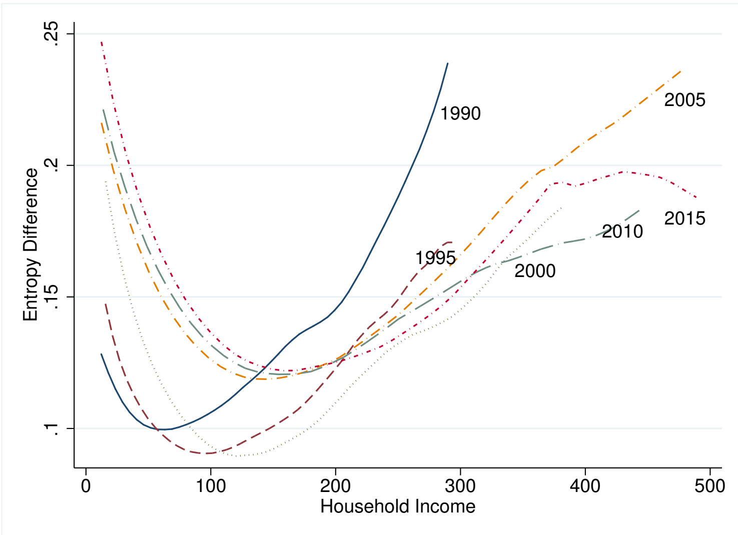

Figure 5: Differences between spending diversity on the aggregate and the household level (aggregation level: 3 expenditure categories)

Note: The curves depict the differences $\hat{E} - E_i$ between spending diversity on the aggregate (50 groups) and the household level. The figure shows that these differences tend to rise (fall) in income at high (low) income levels.

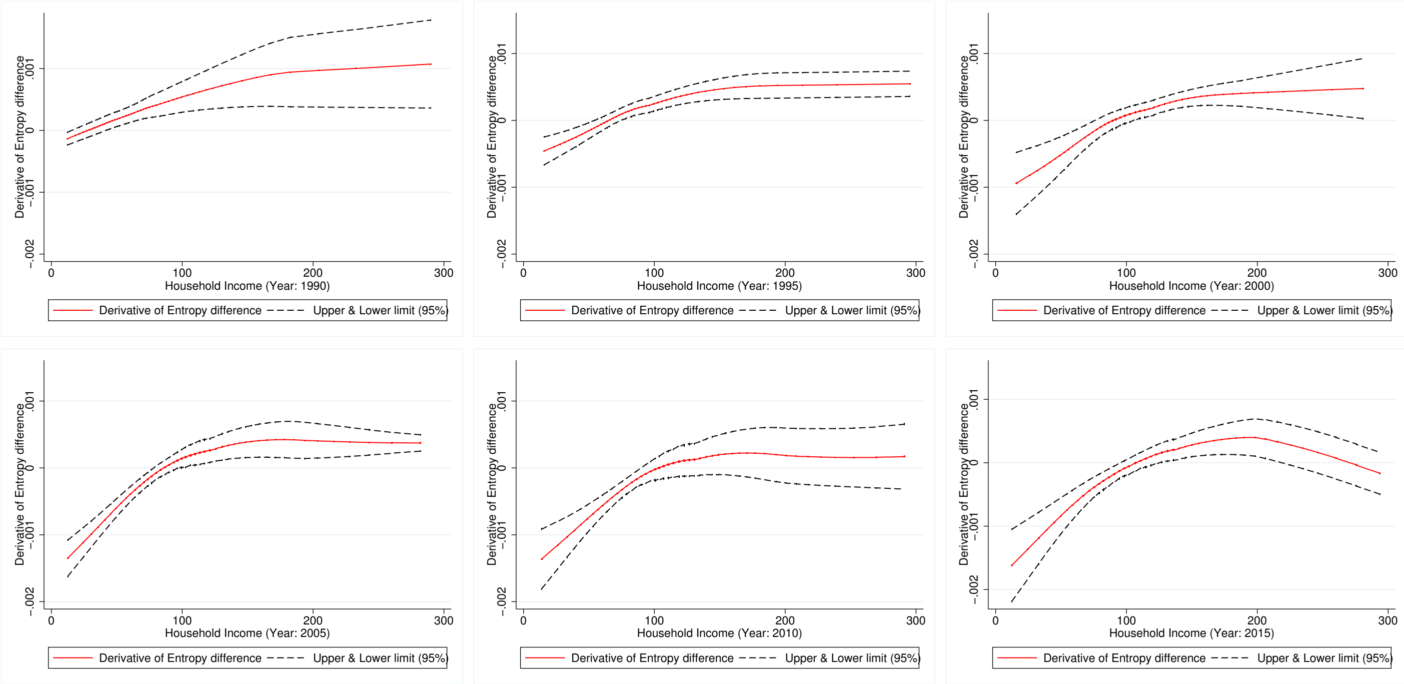

Figure 6: Slopes of the $\hat{E} - E_i$ curves with confidence intervals (aggregation level: 3 expenditure categories)

Notes: The curves depict the slopes $\frac{\partial(\hat{E}-E_i)}{\partial x}$ of the entropy difference curves $(\hat{E} - E_i)$ as a function of income x for the case of 3 categories and 50 groups in the years 1990 , 1995, 2000 , 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

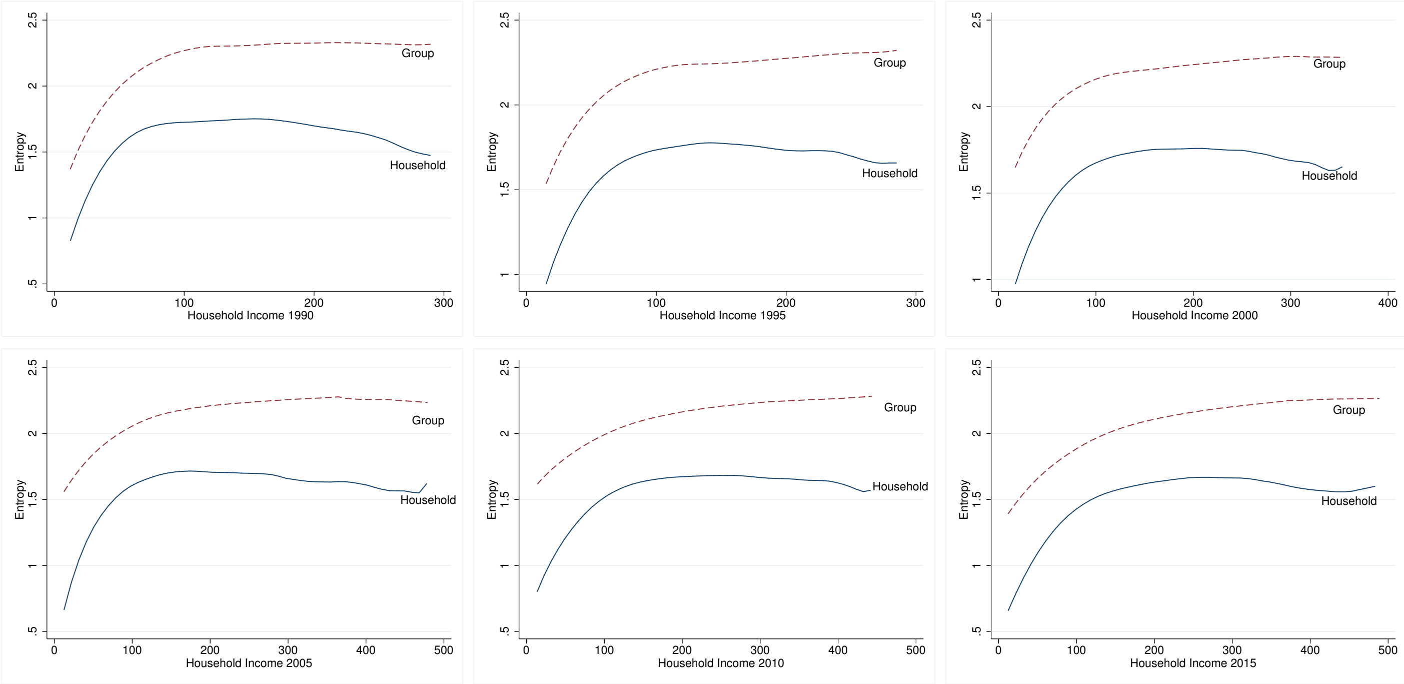

Figure 7: The Engel curves for spending diversity on the household and aggregate (group) level (aggregation level: 12 expenditure categories)

Notes: The Figures depict spending diversity on the household level (solid line, E_i) and on the aggregate level (dashed line, $\hat{E}$) for 1990 (top left), 1995 (top middle), 2000 (top right), 2005 (bottom left), 2010 (bottom middle) and 2015 (bottom right). Expenditure categories are aggregated into 12 categories - see Table 1. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average of the poorest and above the average of the richest income group. As a result, these curves cover a smaller income range than those displayed in Figure 3.

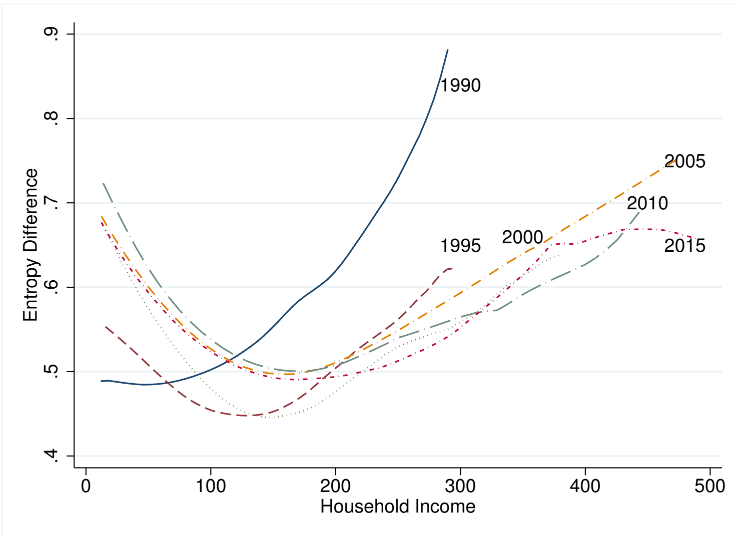

Figure 8: Differences between spending diversity on the aggregate and the household level (aggregation level: 12 expenditure categories)

Note: The differences $\hat{E} - E_i$ between aggregate (group) and household level (individual) entropy of spending.The figure shows that these differences tend to rise (fall) in income at high (low) income levels.

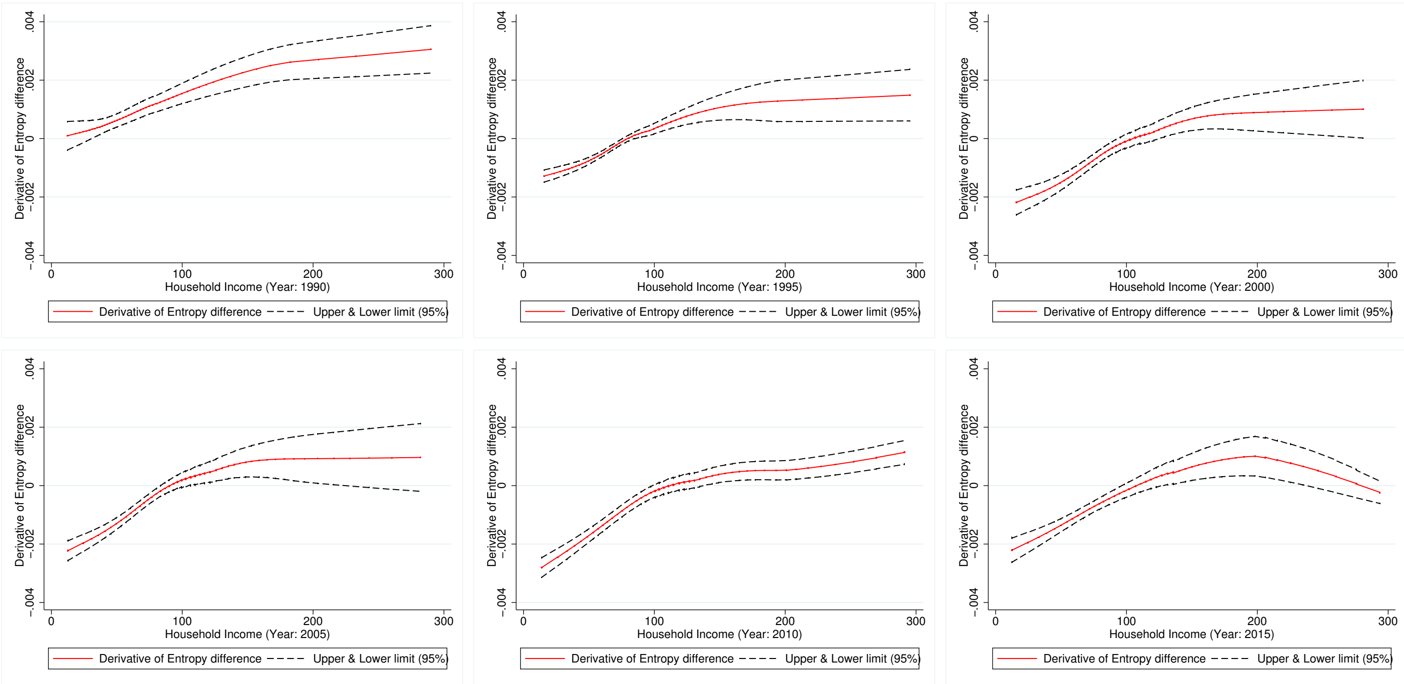

Figure 9: Slopes of the $\hat{E} - E_i$ curves with confidence intervals (aggregation level: 12 expenditure categories)

Notes: The curves depict the slopes $\frac{\partial(\hat{E}-E_i)}{\partial x}$ of the entropy difference curves ($\hat{E} - E_i$) as a function of income x for the case of 12 categories and 50 groups in the years 1990, 1995, 2000, 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

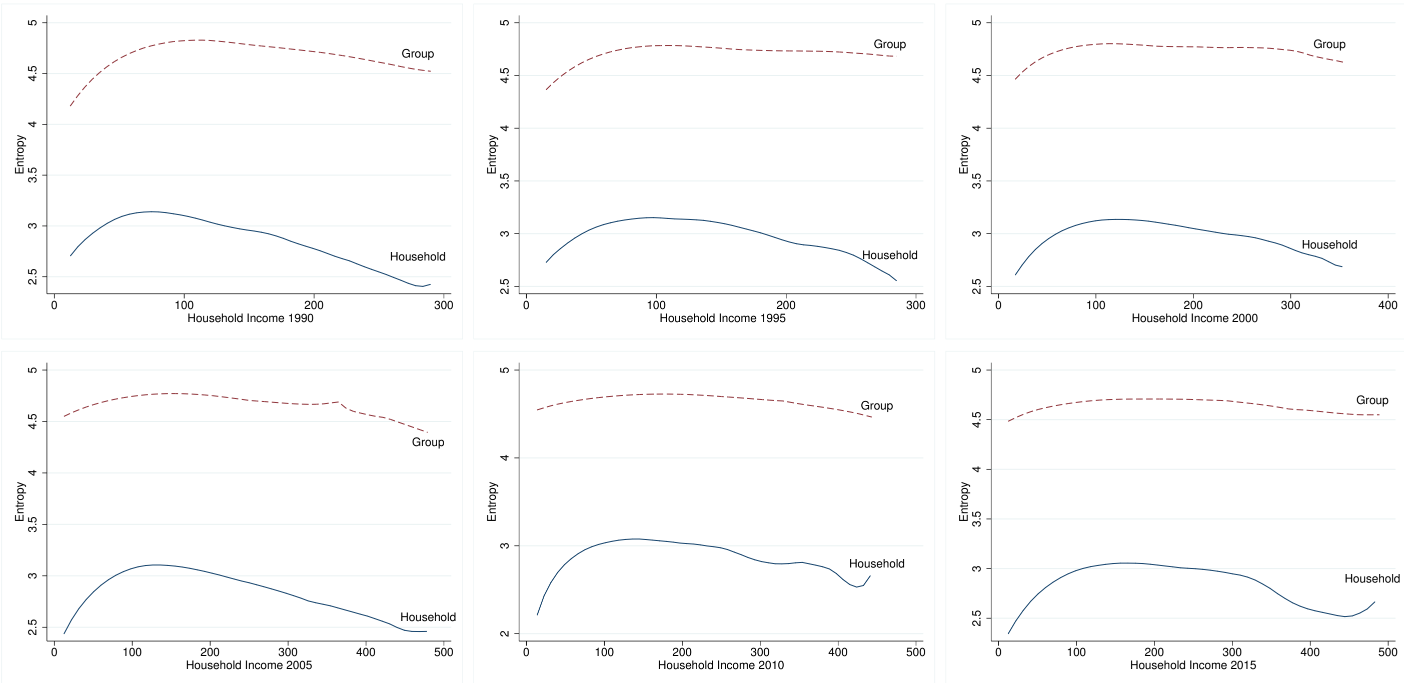

Figure 10: The Engel curves for spending diversity on the household and aggregate (group) level (aggregation level: 200+ expenditure categories)

Notes: The Figures depict spending diversity on the household level (solid line, E_i) and on the aggregate level (dashed line, $\hat{E}$) for 1990 (left), 1995 (middle) and 1995 (right). Expenditure categories were not aggregated. Households are aggregated into 50 representative groups with similar income levels. Note that the individual spending diversity curves are shortened to omit observations below the average of the poorest and above the average of the richest income group. As a result, these curves cover a smaller income range than those displayed in Figure 3.

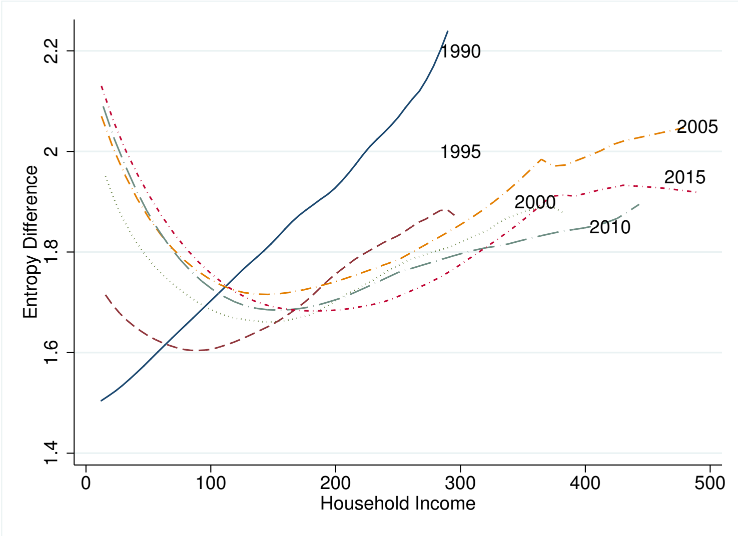

Figure 11: Differences between spending diversity on the aggregate and household level (aggregation level: 200+ Categories)

Note: The difference $\hat{E} - E_i$ between aggregated (group) level and individual (household) level entropy of spending. The figure shows that these differences tend to rise (fall) in income at high (low) income levels.

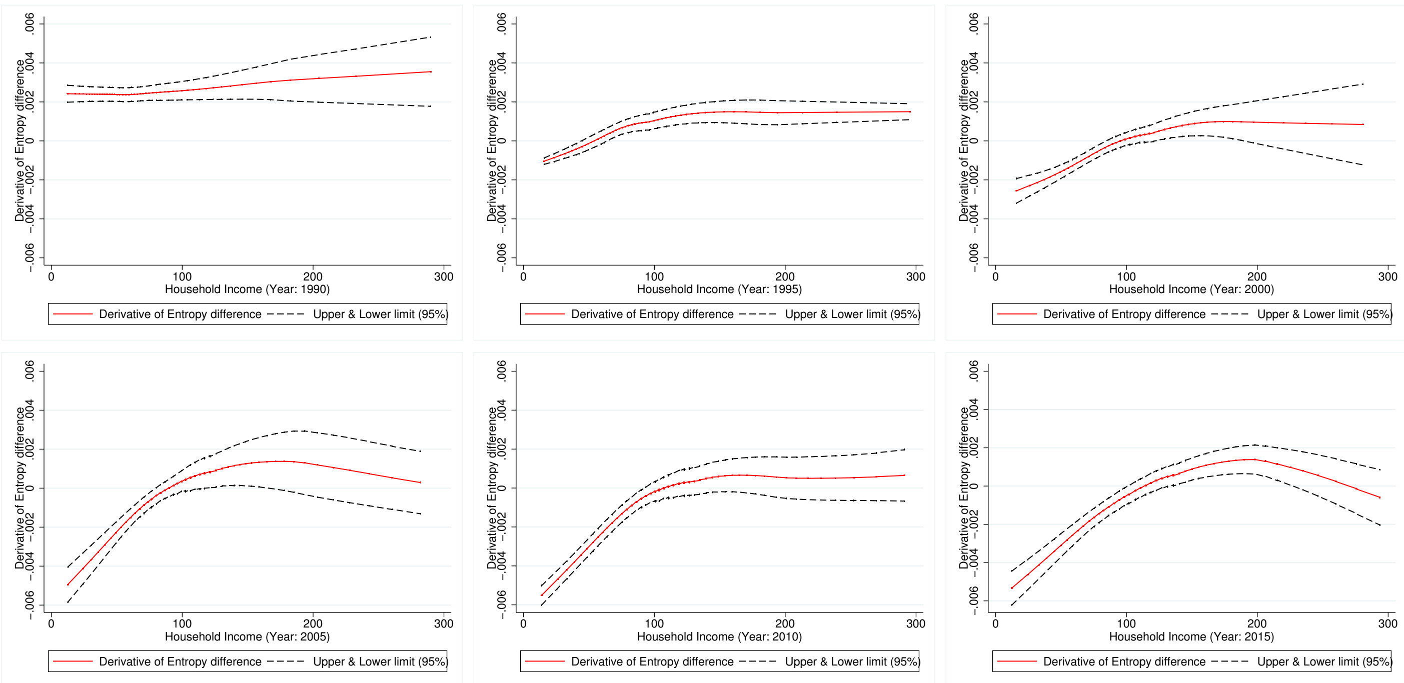

Figure 12: Slopes of the $\hat{E} - E_i$ curves with confidence intervals

Notes: The curves depict the slopes $\frac{\partial(\hat{E}-E_i)}{\partial x}$ of the entropy difference curves $(\hat{E} - E_i)$ as a function of income x for the case of 200+ categories and 50 groups in the years 1990, 1995, 2000, 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

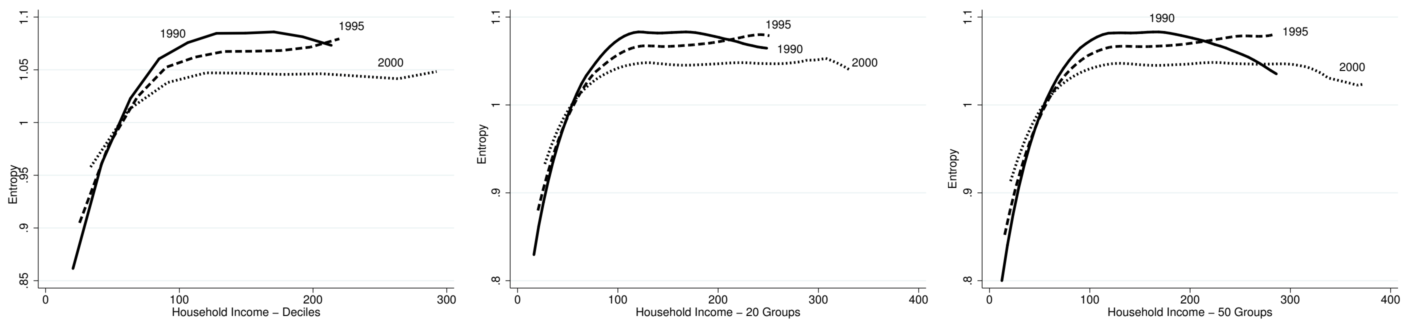

Figure 13: The Engel curve for spending diversity on the aggregate level

Notes: This Figure depicts the entropy $\hat{E}$ of spending on the aggregate level across different household aggregation levels. The graph on the left depicts the case of 10 representative (decile) income groups, the one in the middle the case of 20 groups, and the one on the right the case of 50 income groups.

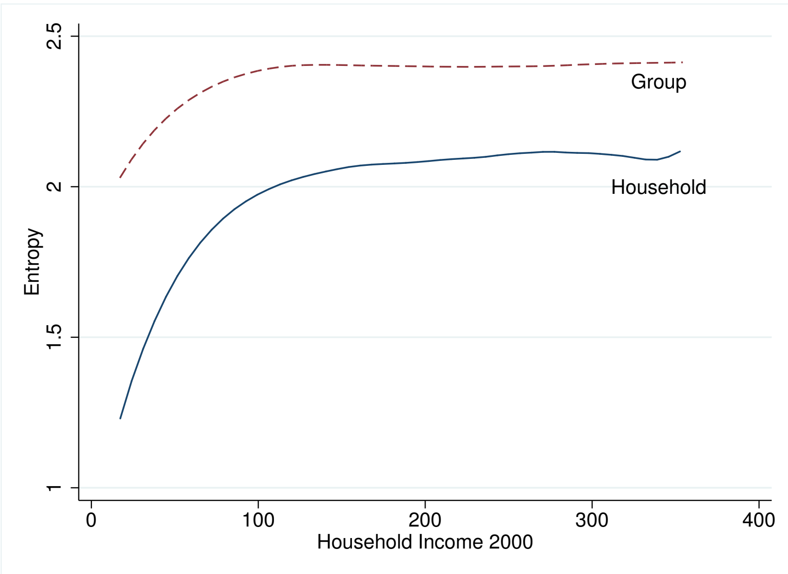

Figure 14: Diversity of variety demand

Note: This figure reports how evenly the consumed varieties are distributed across the 12 categories (see Table 1). This diversity of variety demand is depicted at the level of individual households and at the group level (50 groups) for the year 2000.

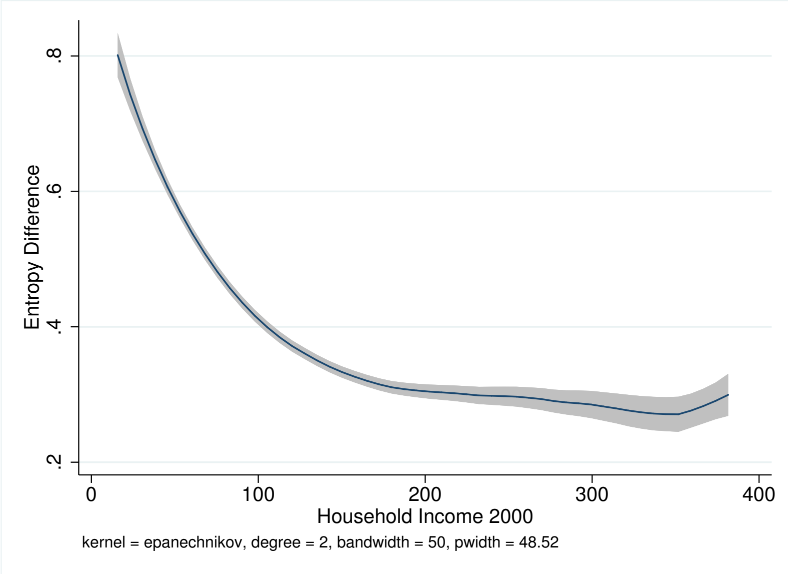

Figure 15: Estimated differences with confidence intervals

Notes: This figure depicts the estimated differences in the diversity of variety demand between aggregate level (50 groups) and household level spending diversity for 2000. The dashed lines represent 95% confidence intervals

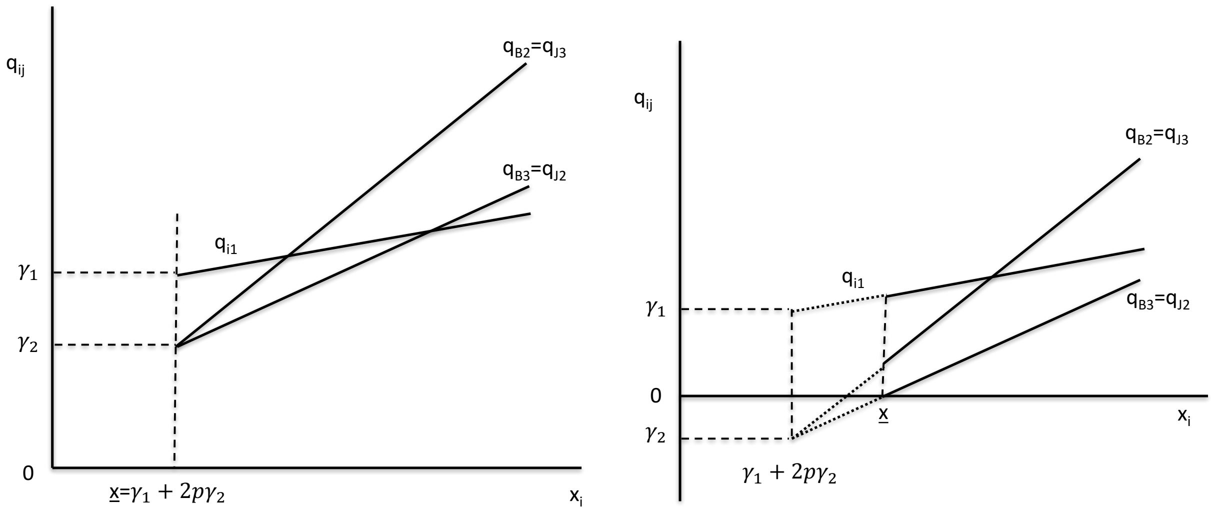

Figure 16: Engel curves in the case of three goods

Notes: The Figures show the Engel curves (i.e. the quantities q_{ij} as a function of the income x_i) arising under the particular assumptions made in the three good example from Section 3.1. The Figure on the left depicts the case where $\gamma_2 > 0$ holds and the figure on the right the case where $\gamma_2 < 0$ holds. In the latter case, the Engel curves are only drawn for income levels $x_i > \underline{x}$ for which households consume positive quantities of all goods.

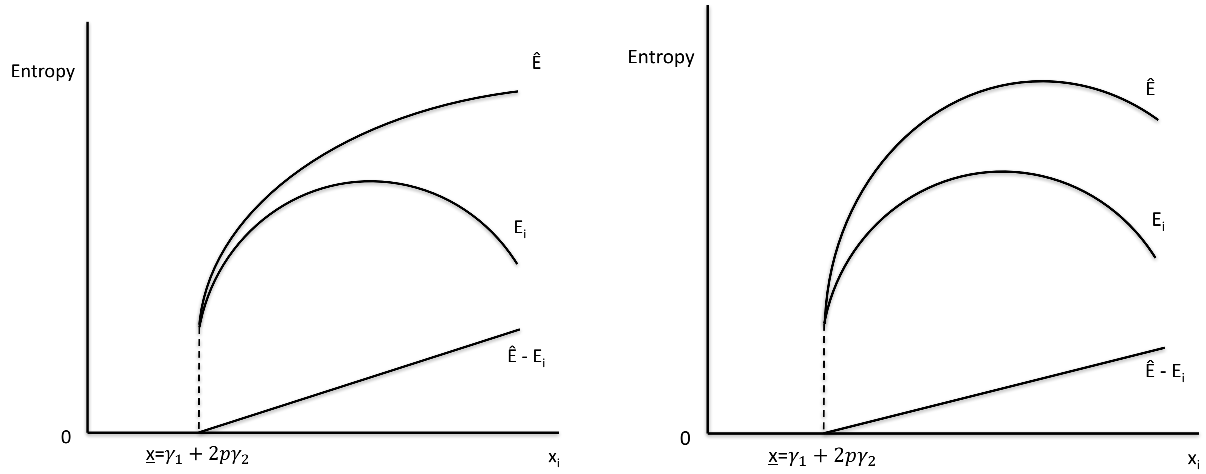

Figure 17: Engel curves for spending diversity on the aggregate and individual household level when $\gamma_2 > 0$

Notes: The Figures show the shapes of the Engel curves for spending diversity that the model can generate when $\gamma_2 > 0$ holds. While the case is considered in which there is always an inverse-U relation between household consumption entropy E_i and household income x_i, there can either be a positive relation between the entropy $\hat{E}$ of spending on the aggregate level and income x_i (Figure on the left) or an inverse-U relation between $\hat{E}$ and x_i (Figure on the right). The entropy difference $\hat{E} - E_i$ rises in x_i in both cases.

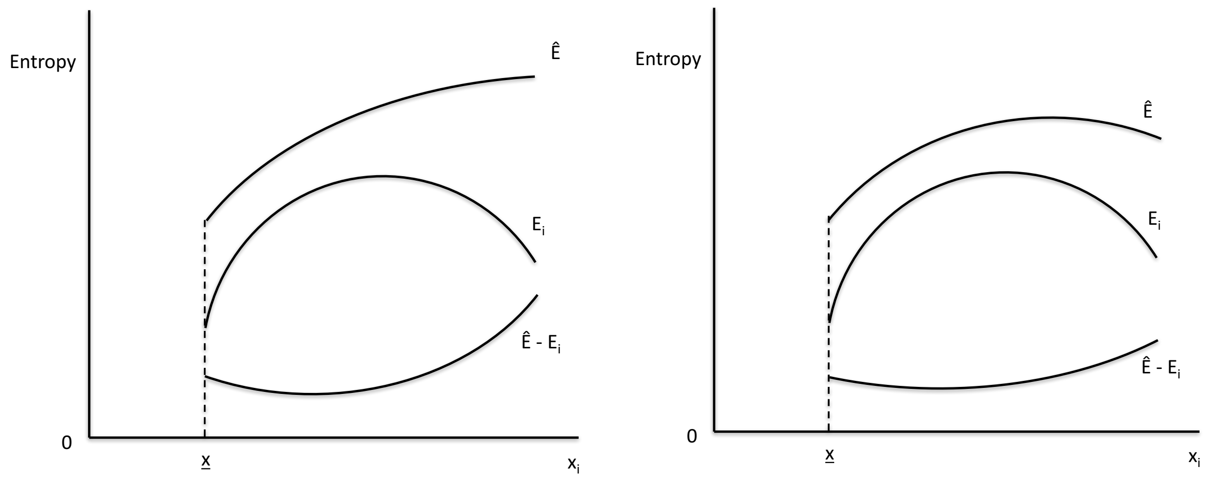

Figure 18: Engel curves for spending diversity on the aggregate and individual household level when $\gamma_2 < 0$

Notes: The Figures show the shapes of the Engel curves for spending diversity that the model can generate when $\gamma_2 < 0$ holds. While the case is considered in which there is always an inverse-U relation between household consumption entropy E_i and household income x_i, there can either be a positive relation between the entropy $\hat{E}$ of spending on the aggregate level and income x_i (Figure on the left) or an inverse-U relation between $\hat{E}$ and x_i (Figure on the right). The entropy difference $\hat{E} - E_i$ first falls and then rises in x_i in both cases.

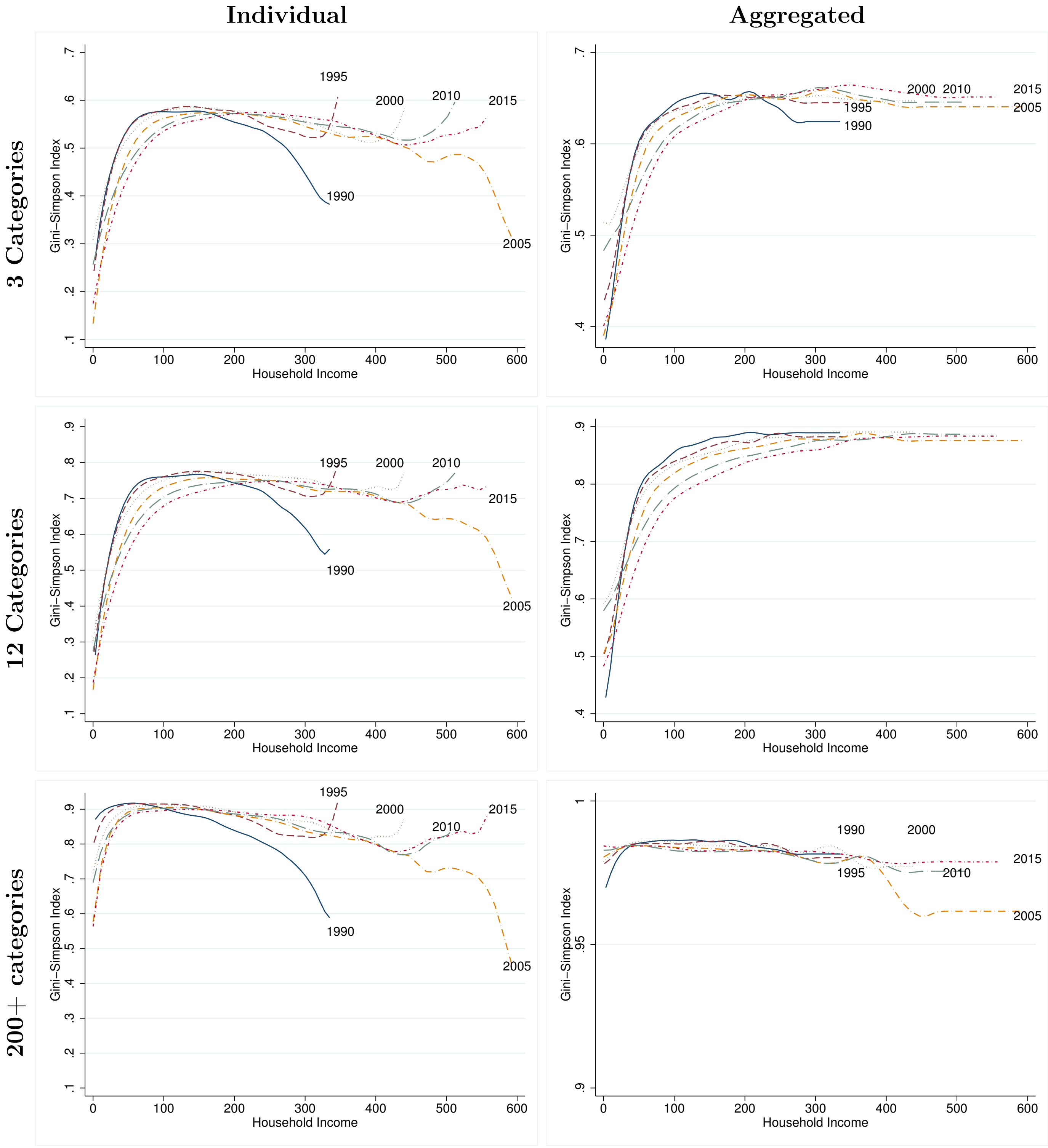

Figure 19: Engel curves for spending diversity (Gini-Simpson Index)

Notes: The Gini-Simpson diversity index is defined as $D_{GS} = \sum_{k=1}^{K} s_k (1 - s_k)$ where s_k is the share of total expenditures allocated to category k in total K categories.

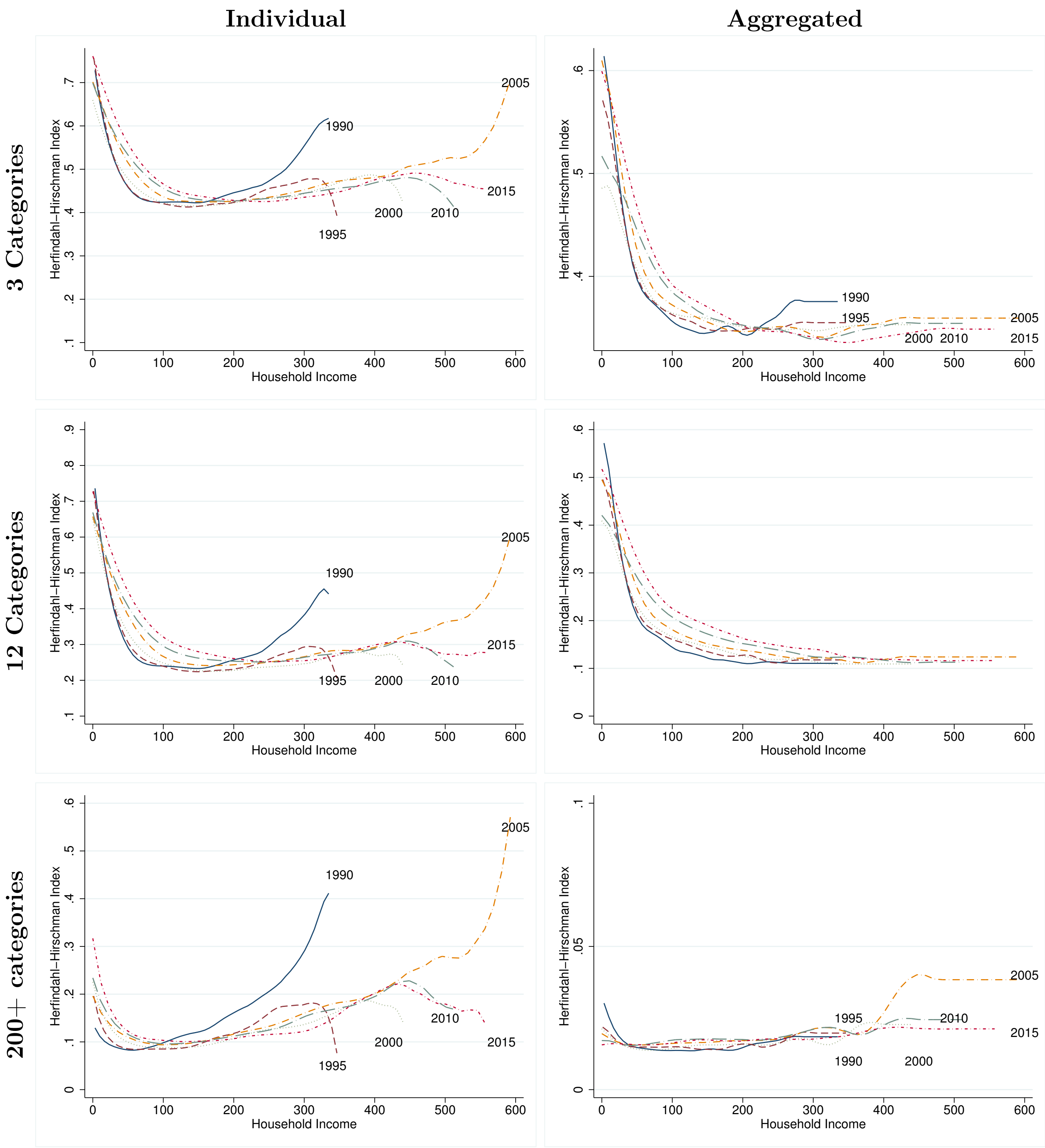

Figure 20: Engel curves for spending diversity (Herfindahl-Hirschman Index)

Notes: The Herfindahl-Hirschman diversity index is defined as $D_H = \sum_{k=1}^{K} s_k^2$ where s_k is the share of total expenditures allocated to category k in total K categories. D_H is equal to 1 if all household expenditure is dedicated to a single expenditure category and falls when expenditures are allocated more evenly across categories. Thus, a lower score of D_H means a higher diversity.

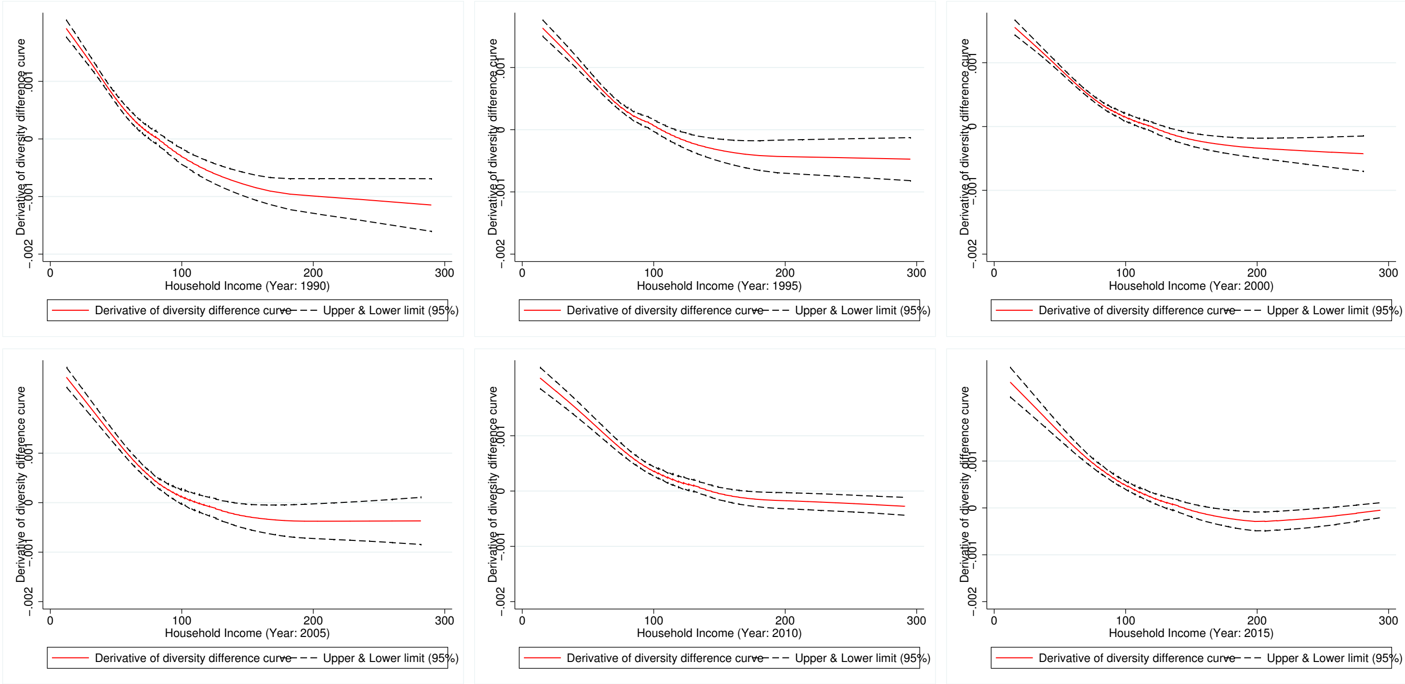

Figure 21: Slopes of diversity difference curves when diversity is measured by the Gini Simpson Index (3 expenditure categories)

Notes: The curves depict the slopes $\frac{\partial(\hat{G}-G_i)}{\partial x}$ of the diversity difference curves ($\hat{G}-G_i$, where G stands for Gini Simpson Index) as a function of income x for the case of 3 categories and 50 groups in the years 1990, 1995, 2000, 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

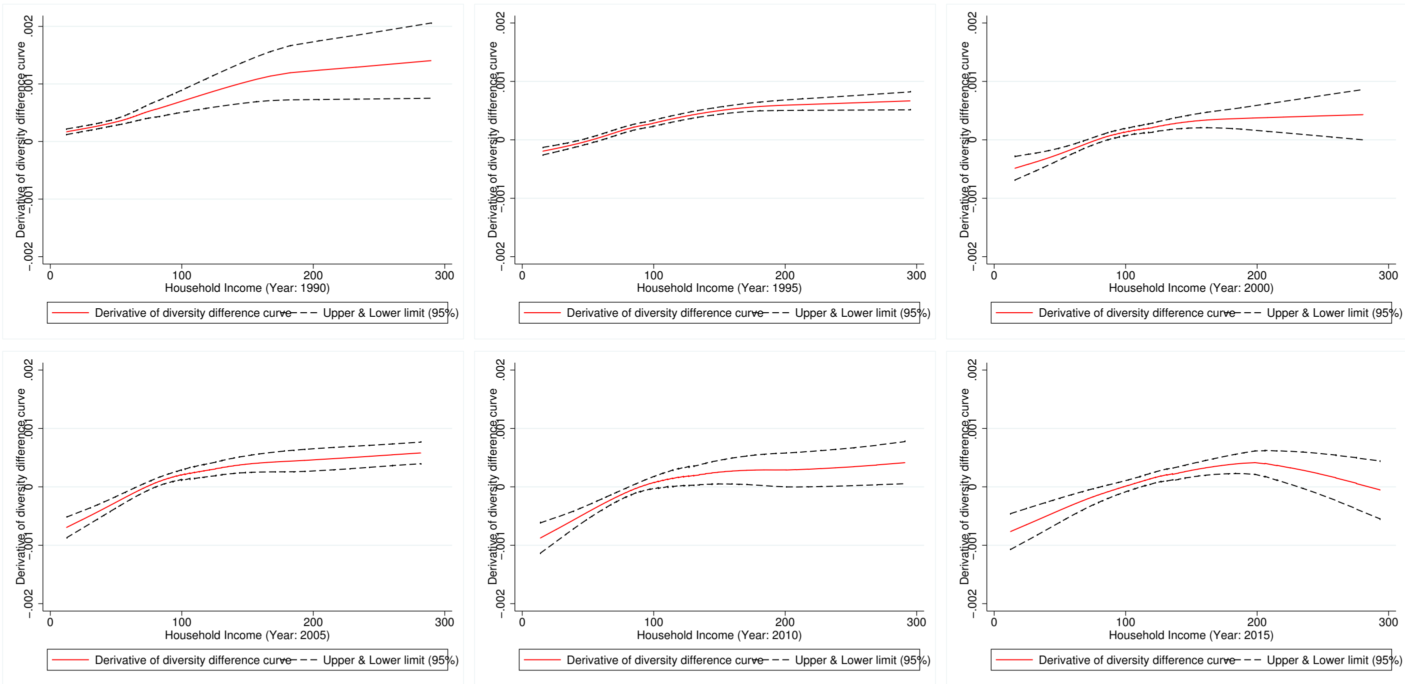

Figure 22: Slopes of diversity difference curves when diversity is measured by Gini Simpson Index (12 expenditure categories)

Notes: The curves depict the slopes $\frac{\partial(\hat{G}-G_i)}{\partial x}$ of the diversity difference curves ($\hat{G}-G_i$, where G stands for Gini Simpson Index) as a function of income x for the case of 12 categories and 50 groups in the years 1990, 1995, 2000, 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

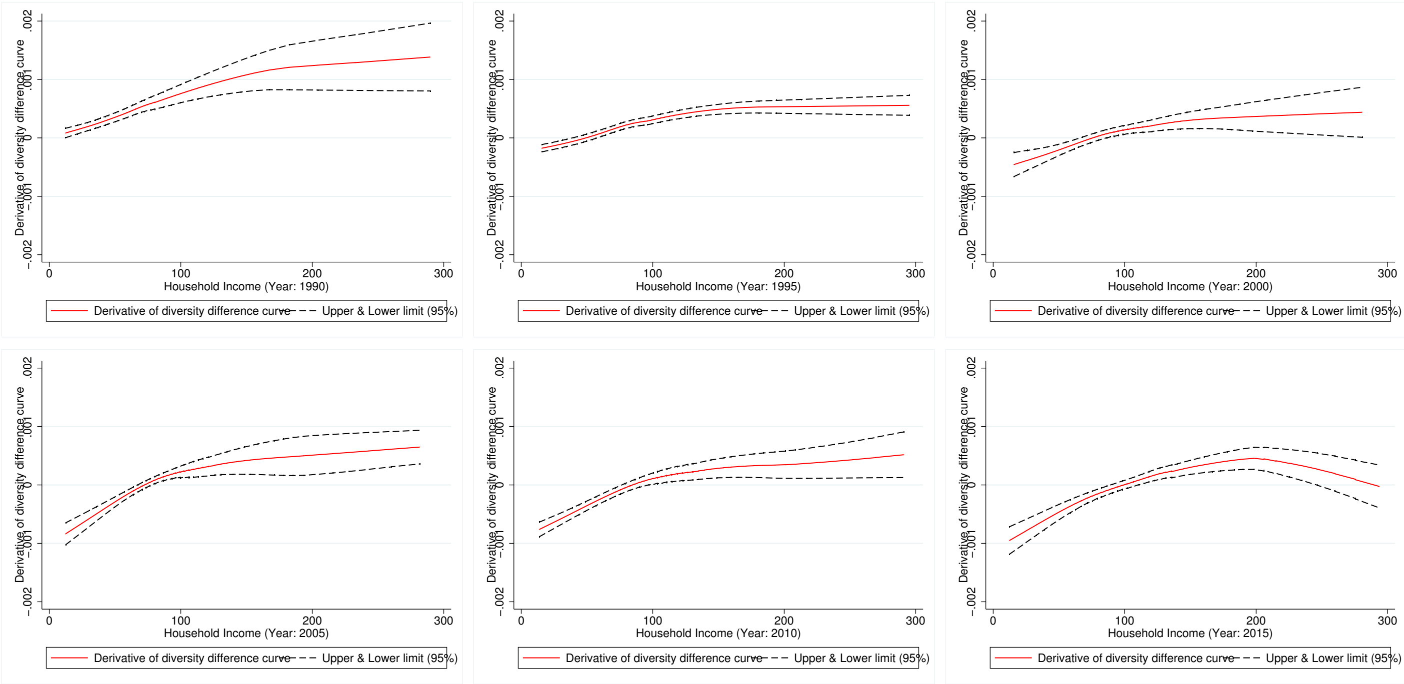

Figure 23: Slopes of diversity difference curves when diversity is measured by Gini Simpson Index (200+ expenditure categories)

Notes: The curves depict the slopes $\frac{\partial(\hat{G}-G_i)}{\partial x}$ of the diversity difference curves ($\hat{G}-G_i$, where G stands for Gini Simpson Index) as a function of income x for the case of 200 categories and 50 groups in the years 1990, 1995, 2000, 2005, 2010 and 2015. The dashed lines represent 95% confidence intervals.

Appendix A: Proofs

A1: Proof of Lemma 1

Proof. Differentiating equation 8, we obtain that $\frac{\partial(s_{i1}(x))}{\partial x} = \frac{2p\gamma_2\left(1-\bar{\beta}\right)-\gamma_1\bar{\beta}p^{1-\varepsilon}}{x^2\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)}$, so that $\frac{\partial(s_{i1}(x))}{\partial x} < 0$ holds when $\gamma_1 > \frac{2\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}}{\bar{\beta}}$ (Condition B). Differentiating equation 12 gives

$$\begin{aligned}\frac{\partial \hat{E}}{\partial x} = -\frac{\partial \hat{s}_1}{\partial x}\left(\ln \hat{s}_1 + 1\right) - 2\frac{\partial \hat{s}_2}{\partial x}\left(\ln \hat{s}_2 + 1\right)\\ = -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1}\right) - \left(\frac{\partial s_{B2}}{\partial x} + \frac{\partial x_{B3}}{\partial x}\right)\left(\ln \hat{s}_2\right)\\ = -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1} - \ln \hat{s}_2\right) = -\frac{\partial s_{B1}}{\partial x}\left(\ln s_{B1} - ln\left(\frac{1-s_{B1}}{2}\right)\right)\end{aligned}$$

where the conditions $\hat{s}_1 = s_{B1}$, $\hat{s}_2(x) = \frac{s_{B2}(x)+s_{B3}(x)}{2}$, $s_{B1} + s_{B2} + s_{B3} = 1$ and $\frac{\partial s_{B1}}{\partial x} + \frac{\partial s_{B2}}{\partial x} + \frac{\partial x_{B3}}{\partial x} = 0$ were used for the transformations. As $\frac{\partial s_{B1}}{\partial x} < 0$, $sign\frac{\partial \hat{E}}{\partial x} = sign\left(\ln s_{B1} - \ln\left(\frac{1-s_{B1}}{2}\right)\right) = sign\left(s_{B1} - \frac{1}{3}\right)$ holds. As $\lim_{x\to\infty} s_{B1} = \frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}}$ and as (by assumption) s_{i1} continuously falls in x, $\frac{\partial \hat{E}}{\partial x} > 0$ therefore always holds if $\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}}{p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}} > \frac{1}{3}$ holds, i.e. if $\bar{\beta} < \frac{2}{p^{1-\varepsilon}+2}$ holds. If $\bar{\beta} > \frac{2}{p^{1-\varepsilon}+2}$, $\frac{\partial \hat{E}}{\partial x} < 0$ holds for large values of x (i.e. $x > \check{x}$), while $\frac{\partial \hat{E}}{\partial x} > 0$ still holds for lower values ($\underline{x} < x < \check{x}$) when $s_{B1}|_{x=\underline{x}} > \frac{1}{3}$ (**Condition C**) holds. The minimum income level $\underline{x}$ is given by $\underline{x} = \gamma_1 + 2p\gamma_2$ when $\gamma_2 = \gamma_3 \geq 0$ holds (as this income is required to purchase the positive subsistence consumption level of each good) and by $\underline{x} = \gamma_1 + 2p\gamma_2 - \gamma_2\left[\frac{\left(1-\bar{\beta}\right)p^{\varepsilon}+\bar{\beta}p}{\bar{\beta}-\beta_{12}}\right]$ when $\gamma_2 = \gamma_3 < 0$ (in this case, $\underline{x}$ is pinned down by the condition $s_{B3}(\underline{x}) = s_{J2}(\underline{x}) = 0$). This implies that $s_{B1}|_{x=\underline{x}} = \frac{\gamma_1}{\gamma_1+2p\gamma_2}$ when $\gamma_2 \geq 0$, and $s_{B1}|_{x=\underline{x}} = \frac{\left(\bar{\beta}-\beta_{B2}\right)\gamma_1-\left(1-\bar{\beta}\right)\gamma_2}{\left(\bar{\beta}-\beta_{B2}\right)\gamma_1-\gamma_2\left(\left(1-\bar{\beta}\right)p^{\varepsilon}+2p\beta_{B2}-p\bar{\beta}\right)}$ when $\gamma_2 < 0$. Plugging these values into Condition C, we obtain that Condition C is satisfied if either $\gamma_1 > p\gamma_2 \geq 0$ (**Condition C1**) holds, or if $\gamma_1 > \frac{-\gamma_2\left(p\left(2\beta_{B2}-\bar{\beta}\right)-\left(1-\bar{\beta}\right)(3-p^{\varepsilon})\right)}{2\left(\bar{\beta}-\beta_{B2}\right)}$ and $\gamma_2 < 0$ (**Condition C2**) holds. □

A2: Proof of Proposition 1

Proof. Differentiating equations 9 and 10 gives $\frac{\partial(s_{B2}(x))}{\partial x} = \frac{\gamma_1\beta_{B2}-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}-p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$ and $\frac{\partial(s_{B3}(x))}{\partial x} = \frac{\gamma_1\left(\bar{\beta}-\beta_{B2}\right)-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}+p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)}{x^2\left(\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon-1}\right)}$, implying that $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ holds when $\gamma_1 > -2\gamma_2 p$ (**Condition D**) holds. As $s_{i1} + s_{i2} + s_{i3} = 1$ and therefore $\frac{\partial(s_{i1})}{\partial x} + \frac{\partial(s_{i2})}{\partial x} + \frac{\partial(s_{i3})}{\partial x} = 0$, the conditions $\frac{\partial(s_{i1}(x))}{\partial x} < 0$ (implied by Condition B) and $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$ imply that $\frac{\partial(s_{B2}(x))}{\partial x} > 0$ needs to hold. The derivative $\frac{\partial(s_{B3}(x))}{\partial x}$ can be either positive or negative, where the latter is only possible if $\gamma_2 > 0$ holds ($\frac{\partial(s_{B3}(x))}{\partial x}$ falls in β_{B2} and is therefore most likely negative when $\beta_{B2} = \bar{\beta}$ holds. As $sign \left.\frac{\partial(s_{B3}(x))}{\partial x}\right|_{\beta_{B2}=\bar{\beta}} = sign\left\{-\gamma_2\right\}$, $\frac{\partial(s_{B3}(x))}{\partial x} < 0$ can only hold if $\gamma_2 > 0$).

Subtracting equation 11 from equation 12 and differentiating with respect to x, we obtain that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ holds when the following **Condition E** is satisfied:

$$\frac{\partial\left(s_{B2}(x)\right)}{\partial x}\left(\ln s_{B2}-\ln \hat{s}_2\right) > \frac{\partial\left(s_{B3}(x)\right)}{\partial x}\left(\ln \hat{s}_2-\ln s_{B3}\right)$$

As $\hat{s}_2(x) = \frac{s_{B2}(x)+s_{B3}(x)}{2}$ and $s_{B2}(x) > s_{B3}(x)$, the terms in brackets are positive, implying that Condition E always holds when $\frac{\partial(s_{B3}(x))}{\partial x} < 0$ holds. When $\frac{\partial(s_{B3}(x))}{\partial x} < 0$, which is only possible if $\gamma_2 > 0$ (see above), $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ therefore holds. In the following, the remaining case where $\frac{\partial(s_{B3}(x))}{\partial x} > 0$ holds is considered. This is done by rewriting Condition E as follows:

$$Z \equiv \frac{\frac{\partial(s_{B2}(x))}{\partial x}}{\frac{\partial(s_{B3}(x))}{\partial x}} > Q \equiv \frac{\ln \hat{s}_2-\ln s_{B3}}{\ln s_{B2}-\ln \hat{s}_2} \tag{13}$$

Due to the concavity of the ln function, $Q > 1$ holds. The proposition studies the case in which $\frac{\partial(s_{B2}(x))}{\partial x} > \frac{\partial(s_{B3}(x))}{\partial x}$, i.e. in which $Z > 1$ holds. The reason for this is that in the case where $\frac{\partial(s_{B2}(x))}{\partial x} < \frac{\partial(s_{B3}(x))}{\partial x}$, $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds for all values of x, which would not be in line with the empirical observations. Inserting the corresponding expressions, Z can be derived as:

$$Z = \frac{\gamma_1\beta_{B2}-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}-p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)}{\gamma_1\left(\bar{\beta}-\beta_{B2}\right)-\gamma_2\left(1-\bar{\beta}\right)p^{\varepsilon}+p\gamma_2\left(\bar{\beta}-2\beta_{B2}\right)} \tag{14}$$

Z is therefore independent of income x. The proof (for the case in which $\frac{\partial(s_{B3}(x))}{\partial x} > 0$) proceeds as follows: In part i) it is shown that $sign\frac{\partial Q}{\partial x} = sign\gamma_2$. In part ii) it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ always holds when $\gamma_2 > 0$ and the case where $\gamma_2 = 0$ is discussed. In part iii), the case where $\gamma_2 < 0$ is analyzed and it is shown that $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ ($Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$) then holds if $\tilde{x} < x < \infty$ ($\underline{x} < x < \tilde{x}$).

i) Deriving Q with respect to x yields

$$\begin{aligned} sign\frac{\partial Q}{\partial x} &= \\ &= sign\left\{\frac{\partial s_{B2}}{\partial x}\left[\frac{1}{s_{B2}+s_{B3}}\left(\ln s_{B2}-\ln s_{B3}\right)-\frac{1}{s_{B2}}\left(\ln\left(\frac{s_{B2}+s_{B3}}{2}\right)-\ln s_{B2}\right)\right]\right. \\ &\quad \left.+\frac{\partial s_{B3}}{\partial x}\left[\frac{1}{s_{B2}+s_{B3}}\left(\ln s_{B2}-\ln s_{B3}\right)-\frac{1}{s_{B3}}\left(\ln s_{B2}-\ln\left(\frac{s_{B2}+s_{B3}}{2}\right)\right)\right]\right\} \end{aligned}$$

Bringing all terms to a common denominator gives

$$\begin{aligned} sign\frac{\partial Q}{\partial x} &= \\ &= sign\left\{s_{B2}\ln s_{B2}+s_{B3}\ln s_{B3}-2\left(\frac{s_{B2}+s_{B3}}{2}\right)\ln\left(\frac{s_{B2}+s_{B3}}{2}\right)\right\}\left[s_{B3}\frac{\partial s_{B2}}{\partial x}-s_{B2}\frac{\partial s_{B3}}{\partial x}\right] \end{aligned}$$

As the term in curly brackets is equal to $\hat{E}(x)-E_i(x)$ and therefore positive (see above),

$$sign\frac{\partial Q}{\partial x} = sign\left[s_{B3}\frac{\partial s_{B2}}{\partial x}-s_{B2}\frac{\partial s_{B3}}{\partial x}\right]$$

Inserting $\frac{\partial s_{B2}}{\partial x} = \frac{p\beta_{B2} - s_{B2}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ and $\frac{\partial s_{B3}}{\partial x} = \frac{p\left(\bar{\beta}-\beta_{B2}\right) - s_{B3}\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}{x\left(p\bar{\beta}+\left(1-\bar{\beta}\right)p^{\varepsilon}\right)}$ (note that both derivatives are assumed to be positive here) and then s_{B2} and s_{B3}, we get that

$$
\begin{aligned}
sign\frac{\partial Q}{\partial x} &= sign\left[\beta_{B2}s_{B3} - \left(\bar{\beta}-\beta_{B2}\right)s_{B2}\right] \\
&= sign\left\{\gamma_2\left[\left(1-\bar{\beta}\right)p^{\varepsilon}\left(2\beta_{B2}-\bar{\beta}\right) + p\left(\left(\beta_{B2}\right)^2 - \left(\bar{\beta}-\beta_{B2}\right)^2\right)\right]\right\} = sign\gamma_2
\end{aligned}
$$

ii) When $\gamma_2 > 0$, Q continuously rises in x. $Z > Q$ therefore holds for all values of x if it holds for $x \to \infty$ (Z does not depend on x). Inserting the corresponding budget shares into the expression of Q and solving for the limit gives

$$
\lim_{x\to\infty} Q = \frac{\ln\left(\frac{\bar{\beta}}{2}\right) - \ln\left(\bar{\beta}-\beta_{B2}\right)}{\ln\beta_{B2} - \ln\left(\frac{\bar{\beta}}{2}\right)}
$$

As Z continuously rises in γ_2 (using equation 14 it can be shown that $sign\frac{\partial Z}{\partial \gamma_2} = sign\left(2\beta_{B2}-\bar{\beta}\right)\left(\left(1-\bar{\beta}\right)p^{\varepsilon}+p\bar{\beta}\right) > 0$), $Z > Q$ therefore always holds if $Z|_{\gamma_2=0} = \frac{\beta_{B2}}{\bar{\beta}-\beta_{B2}} > \lim_{x\to\infty} Q$ holds. This inequality is satisfied if $\beta_{B2}\ln\beta_{B2} + \beta_{J2}\ln\beta_{J2} = E_i > 2\frac{\bar{\beta}}{2}\ln\left(\frac{\bar{\beta}}{2}\right) = \hat{E}$ holds. $E_i > \hat{E}$ holds if $\beta_{B2} > \frac{\bar{\beta}}{2}$ and $x > \underline{x}$ (see footnote 22). Consequently, $Z > Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ hold when $\gamma_2 > 0$. When $\gamma_2 = 0$, $Q = \frac{\ln\left(\frac{\bar{\beta}}{2}\right)-\ln\left(\bar{\beta}-\beta_{B2}\right)}{\ln\beta_{B2}-\ln\left(\frac{\bar{\beta}}{2}\right)}$ holds independently of x, so that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ still holds for $x > \underline{x}$. At the point where $\gamma_2 = 0$ and $x = \underline{x}$, $s_{i2}(x) = s_{i3}(x)$ and therefore $E_i = \hat{E}$, implying that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} = 0$.

iii) When $\gamma_2 < 0$, $Z > Q$ still holds when x is sufficiently large (see part *ii*) of the proof), implying that $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ still holds in this case. When $\gamma_2 < 0$ and x approaches the lower bound $\underline{x}$, $s_{B3} = s_{J2}$ approaches zero (while $s_{B2} = s_{J3}$ remains positive), implying that $Q \equiv \frac{\ln\hat{s}_2 - \ln s_{B3}}{\ln s_{B2} - \ln\hat{s}_2}$ becomes infinitely large and that $Z < Q$ and therefore $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ holds. As Q continuously falls in x when $\gamma_2 < 0$ holds (see part *i*) of the proof), $Z < Q$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} < 0$ therefore holds in this case when $\underline{x} \leq x < \tilde{x}$ and $Z > Q$ and $\frac{\partial\left(\hat{E}(x)-E_i(x)\right)}{\partial x} > 0$ when $\tilde{x} < x < \infty$, where $\tilde{x}$ ($\underline{x} < \tilde{x} < \infty$) is a positive parameter. □

A3: Proof of Proposition 2

Proof. Using equation 2 and inserting the optimal quantities q^*_{i1}, q^*_{i2}, and q^*_{i3} (that can be obtained from equations 8, 9 and 10), the utility $U_i(3)$ that household i obtains from consuming positive quantities of all three goods can be derived as:

$$
\begin{aligned}
&U_i(3) = \\
&\left[\left(1-\bar{\beta}\right)\left(q^*_{i1}(x_i)-\gamma_1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q^*_{i2}(x_i)-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}\left(q^*_{i3}(x_i)-\gamma_2\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&\qquad = \left(x_i - 2p\gamma_2 - \gamma_1\right)\left[1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right]^{\frac{1}{\varepsilon-1}}
\end{aligned}
$$

This utility is therefore independent of β_{i2} and of preference heterogeneity (it can, moreover, be shown that this is not only holds when $\gamma_2 = \gamma_3$, but also in the more general case where $\gamma_2 \neq \gamma_3$).

When an interior solution for F_i exists, individual i must be indifferent between consuming all three goods and having income x_i, and only consuming Good 1 and having income $x_i + F_i$. In the latter case, utility is given by:

$$U_i(1) = \left[\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}(x_i + F_i - \gamma_1)^{\frac{\varepsilon-1}{\varepsilon}} + (\beta_{i2})^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Setting $U_i(1) = U_i(3)$ allows to solve for the value of variety F_i:

$$F_i =$$

$$\gamma_1 - x_i + \left[\frac{(x_i - 2p\gamma_2 - \gamma_1)^{\frac{\varepsilon-1}{\varepsilon}}\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} - (\beta_{i2})^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}} - \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$\equiv \gamma_1 - x_i + [R]^{\frac{\varepsilon}{\varepsilon-1}} \quad (15)$$

When $\beta_{i2} > \frac{\bar{\beta}}{2}$, household i prefers Good 2 over Good 3 and the critical income level $\underline{x}$ above which the household consumes positive quantities of all goods (if they are available) can be derived from the condition $q^*_{i3}(\underline{x}) = 0$ and is given by:

$$\underline{x} = \gamma_1 + 2p\gamma_2 + (-\gamma_2)\frac{\left(1-\bar{\beta}\right)p^{\varepsilon} + \bar{\beta}p}{\bar{\beta}-\beta_{i2}} \quad (16)$$

The larger β_{i2} is, the larger $\underline{x}$ therefore becomes. As households find it optimal to consume positive quantities of Goods 2 and 3 when $x_i > \underline{x}$, F_i must be positive under this condition. This, however, does not guarantee that there is an interior solution. Indeed, no finite interior F_i exists when $\varepsilon < 1$ and when income x_i is sufficiently high. The reason for this is that when $\varepsilon < 1$, $\underset{x_i\to\infty}{lim} U_i(1)$ is finite, while $\underset{x_i\to\infty}{lim} U_i(3)$ is infinite, implying that even an infinite increase in income F_i cannot compensate a household with a sufficiently high x_i for the loss of the possibility to consume Goods 2 and 3. When $\varepsilon < 1$, F_i goes to infinity when the term R in equation 15 approaches zero (from above), implying that an interior solution for F_i can only exist if R is positive. This is the case if:

$$x_i < \gamma_1 + 2p\gamma_2 + \left[\frac{(\beta_{i2})^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}(-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}}}{\left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \equiv \hat{x}$$

$\hat{x}$ is falling in β_{i2} when $\beta_{i2} > \frac{\bar{\beta}}{2}$ and $\varepsilon < 1$. An interior solution for F_i therefore exists if either $\varepsilon > 1$ and $x_i \geq \underline{x}$, or if $\varepsilon < 1$ and $\underline{x} \leq x_i < \hat{x}$. The latter condition gives a non-empty set of permissible values for x_i when the following inequality is satisfied: $p^{-(1-\varepsilon)} > \frac{(\beta_{i2})^{\frac{1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}}{\left(1-\bar{\beta}\right)\left(\bar{\beta}-\beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}}} - \frac{\bar{\beta}}{\left(1-\bar{\beta}\right)}$. This condition holds given that p is not too large.

a) and b): Deriving F_i from equation 15 with respect to β_{i2} yields:

$$\frac{\partial F_i}{\partial \beta_{i2}} = \frac{1}{(\varepsilon-1)} R^{\frac{1}{\varepsilon-1}} (-\gamma_2)^{\frac{\varepsilon-1}{\varepsilon}} \left(\left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} - (\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} \right) \frac{1}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}}$$

When $\gamma_2 < 0$ and $\beta_{i2} > \frac{\bar{\beta}}{2}$, $\left(\bar{\beta} - \beta_{i2}\right)^{\frac{1-\varepsilon}{\varepsilon}} - (\beta_{i2})^{\frac{1-\varepsilon}{\varepsilon}} > 0$ (< 0) holds when $\varepsilon > 1$ ($\varepsilon < 1$), implying that $\frac{\partial F_i}{\partial \beta_{i2}} > 0$ holds in this case. When $\varepsilon \neq 1$ and $\beta_{i2} = \frac{\bar{\beta}}{2}$, or when $\varepsilon > 1$ and $\gamma_2 = 0$, $\frac{\partial F_i}{\partial \beta_{i2}} = 0$ holds instead.

Due to symmetry, $\frac{dF_i}{d\beta_{i3}} > 0$ holds in the case where $\gamma_2 < 0$ and where $\beta_{i3} > \frac{\bar{\beta}}{2}$ (and $\frac{dF_i}{d\beta_{i3}} = 0$ when $\varepsilon > 1$ and $\gamma_2 = 0$). F_i therefore increases in β_{ij} when $\gamma_2 < 0$ and when $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds.

This result can be generalized to the case where households have access to and consume positive quantities of $N > 1$ instead of one goods absent innovation or trade and where there is preference heterogeneity with respect to two further goods that can be introduced. The value of increasing variety by two goods is then measured by the income F_i that households i would require as a compensation for losing the possibility to consume these new goods. In such an extended setting, F_i can be derived from the equation $U_i(N) = U_i(N+2)$ that equates the utility derived from consuming N goods to that derived from consuming $N+2$ goods, instead of the equation $U_i(1) = U_i(3)$. As $U_i(N)$ and $U_i(N+2)$ can be derived by adding new terms that are independent of β_{i2} inside the brackets of the equations determining $U_i(1)$ and $U_i(3)$, $U_i(N)$ again increases in F_i and depends in the same qualitative way on β_{i2} as $U_i(1)$ does, and $U_i(N+2)$ is again independent of β_{i2}. Consequently, $\frac{\partial F_i}{\partial \beta_{i2}} > 0$ still holds in this case (when $\gamma_2 < 0$ and $\beta_{i2} > \frac{\bar{\beta}}{2}$).

When $\beta_{i2} > \frac{\bar{\beta}}{2}$ and $\varepsilon > 1$, $F_i > F_a > 0$ and therefore $\frac{F_i}{F_a} > 1$ holds when $\underline{x} \leq x_i < \infty$ ($\underline{x}$ indicates the threshold income at which a household with heterogeneous preferences consumes zero units of Good 3 and a positive amount of Good 2 and at which a household with average preferences consumes positive amounts of both of these goods. Consequently, $F_a > 0$ holds in this case). Moreover, using equation 15 to determine $\frac{F_i}{F_a}$, it can be shown that $\lim_{x_i \to \infty} \frac{F_i}{F_a} = 1$. When $\beta_{i2} > \frac{\bar{\beta}}{2}$, $\varepsilon < 1$, and $\underline{x} \leq x_i < \hat{x}$, $\frac{F_i}{F_a} > 1$ again holds, but $\lim_{x_i \to \hat{x}} \frac{F_i}{F_a} = \infty$ in this case. The reason why $\frac{F_i}{F_a}$ goes to infinity is that F_i reaches infinity when $x_i = \hat{x}$, while F_a is still finite at this level and only reaches infinity at a larger level of x_i.

c): Deriving F_i with respect to γ_2 gives

$$\frac{\partial F_i}{\partial \gamma_2} =$$

$$R^{\frac{1}{\varepsilon-1}} \left((\beta_{i2})^{\frac{1}{\varepsilon}} (-\gamma_2)^{-\frac{1}{\varepsilon}} + \left(\bar{\beta} - \beta_{i2}\right)^{\frac{1}{\varepsilon}} (-\gamma_2)^{-\frac{1}{\varepsilon}} - 2p \left(x_i - 2p\gamma_2 - \gamma_1\right)^{-\frac{1}{\varepsilon}} \left(1 - \bar{\beta} + \bar{\beta} p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} \right) \frac{1}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}}$$

$$\equiv R^{\frac{1}{\varepsilon-1}} S \frac{1}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}}$$

The term S is positive when $x_i > \ddot{x} \equiv \gamma_1 + 2p\gamma_2 + (-\gamma_2) \frac{2^{\varepsilon} p^{\varepsilon} \left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)}{\left((\beta_{i2})^{\frac{1}{\varepsilon}} + \left(\bar{\beta}-\beta_{i2}\right)^{\frac{1}{\varepsilon}}\right)^{\varepsilon}}$ holds. As $\ddot{x} < \underline{x}$ (with $\underline{x}$ defined as in equation 16 above when $\beta_{i2} \geq \frac{\bar{\beta}}{2}$), $S > 0$ holds when $x_i > \underline{x}$. As also $R > 0$, we therefore obtain that $\frac{\partial F_i}{\partial \gamma_2} > 0$ when $x_i > \underline{x}$.

d): Deriving F_i with respect to x_i gives

$$\frac{\partial F_i}{\partial x_i} = R^{\frac{1}{\varepsilon-1}} \left((x_i - 2p\gamma_2 - \gamma_1)^{-\frac{1}{\varepsilon}} \left(1 - \bar{\beta} + \bar{\beta} p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} \right) \frac{1}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}} - 1$$

As $R^{\frac{1}{\varepsilon-1}} = (F_i + x_i - \gamma_1)^{\frac{1}{\varepsilon}}$ and $F_i > 0$ when $x_i > \underline{x}$, $\frac{\partial F_i}{\partial x_i} > 0$ always holds if it holds when the term $R^{\frac{1}{\varepsilon-1}}$ in the expression of $\frac{\partial F_i}{\partial x_i}$ is replaced by $(x_i - \gamma_1)^{\frac{1}{\varepsilon}}$, i.e. when $(x_i-\gamma_1)^{\frac{1}{\varepsilon}} \left((x_i - 2p\gamma_2 - \gamma_1)^{-\frac{1}{\varepsilon}} \left(1 - \bar{\beta} + \bar{\beta} p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}} \right) \frac{1}{\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}} - 1 > 0$ (**Condition F**) holds. The condition $x_i > \underline{x}$ implies that $\gamma_2 > \widehat{\gamma_2} \equiv -\frac{(x-\gamma_1)\left(\bar{\beta}-\beta_{i2}\right)}{\left(1-\bar{\beta}\right)p^{\varepsilon} - \bar{\beta}p + 2p\beta_{i2}}$ needs to hold when $\beta_{i2} \geq \frac{\bar{\beta}}{2}$ holds (this can be derived from equation 16). As the term $(x_i - 2p\gamma_2 - \gamma_1)^{-\frac{1}{\varepsilon}}$ rises in γ_2, $\frac{\partial F_i}{\partial x_i} > 0$ therefore always holds if Condition F still (weakly) holds when γ_2 is replaced by its minimal level $\widehat{\gamma_2}$. As the latter is the case when $\beta_{i2} \geq \frac{\bar{\beta}}{2}$, $\frac{\partial F_i}{\partial x_i} > 0$ therefore always holds in this case. Symmetry implies that it also holds when $\beta_{i3} \equiv \bar{\beta} - \beta_{i2} > \frac{\bar{\beta}}{2}$.

Deriving $\frac{\partial F_i}{\partial \beta_{i2}}$ with respect to x_i gives $\frac{\partial^2 F_i}{\partial \beta_{i2} \partial x_i} = \left(\frac{\partial F_i}{\partial \beta_{i2}}\right) R^{-1} \frac{(x_i - 2p\gamma_2 - \gamma_1)^{-\frac{1}{\varepsilon}} \left(1-\bar{\beta}+\bar{\beta}p^{1-\varepsilon}\right)^{\frac{1}{\varepsilon}}}{\varepsilon\left(1-\bar{\beta}\right)^{\frac{1}{\varepsilon}}}$. When $\beta_{i2} \neq \frac{\bar{\beta}}{2}$ and $x_i > \underline{x}$, this cross-derivative is positive as $\frac{\partial F_i}{\partial \beta_{i2}} > 0$, $R > 0$ and $x_i > 2p\gamma_2 - \gamma_1$ hold in this case.

As $\frac{\partial^2 F_i}{\partial \beta_{i2} \partial x_i} = \frac{\partial^2 F_i}{\partial x_i \partial \beta_{i2}} > 0$ when $\beta_{i2} \neq \frac{\bar{\beta}}{2}$ and as $\frac{\partial F_i}{\partial x_i} > 0$, $\frac{\partial F_i}{\partial x_i} > \frac{\partial F_a}{\partial x_i} > 0$ holds ($F_a = F_i \Big|_{\beta_{i2} = \frac{\bar{\beta}}{2}}$).

$\square$